47p

TECHNICAL NOTE R-41

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FORTRAN PROGRAMS FOR PLUG NOZZLE DESIGN

NASA CR 51300

Prepared By

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March, 1963

BROWN ENGINEERING COMPANY INC.
HUNTSVILLE, ALABAMA

OTS PRICE

XEROX $ 4.60ph

MICROFILM $ 1.61

TECHNICAL NOTE R-41

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March, 1963

Prepared For

ADVANCED PROPULSION SECTION
PROPULSION AND MECHANICS BRANCH
P & VE DIVISION
GEORGE C. MARSHALL SPACE FLIGHT CENTER

By

SCIENTIFIC RESEARCH LABORATORIES
BROWN ENGINEERING COMPANY, INC.

Contract No. NAS8-2485

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ABSTRACT

22139

Two FORTRAN computer programs for the design of pure external and internal-external expansion plug nozzles are described in this report. The output from these programs includes the contour of the nozzle and various performance parameters. The approximate design method is based on simple wave flow concepts which are described by T. L. Deyound.

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LIST OF SYMBOLS

A	Surface area of Prandtl-Meyer expansion wave after it is revolved about the plug axis
a	Length of triangle side
b	Length of triangle side
C_F	Thrust coefficient
F	Thrust
f	Function
g	Constant of proportionality in Newton's second law
h_t	Width of throat gap on pure external expansion plug nozzle
Isp	Specific impulse
L	Chord length of internal circular arc contour
M	Mach number
M^*	Ratio of local velocity to velocity at sonic flow conditions
$\dot{m}$	Mass flow rate
N	Number of contour points computed on pure external expansion plug nozzle
N_1	Number of internal contour points computed on internal-external expansion plug nozzle
N_2	Number of external contour points computed on internal-external expansion plug nozzle
n	Any number of the series 0, 1, 2, ... n
P	Static pressure
R	Radius from plug axis

List of Symbols (Cont.)

R_r	Radius of internal circular arc contour
T	Temperature
V	Velocity on expansion wave through the point indicated by subscript
X	Axial distance from lip of shroud

Greek Symbols:

β	Central angle between the radius to point p and that to any point x_1 on the internal circular arc contour of an internal-external expansion plug nozzle
γ	Ratio of specific heats
Δ	Small increment
δ	Angle between plug axis and sonic line on pure external expansion plug nozzle
ε	Expansion ratio
ϕ	Angle between plug axis and Prandtl-Meyer expansion wave
μ	Mach angle
ν	Prandtl-Meyer turning or expansion angle
ϕ	Mass density
θ	Flow angle measured from plug axis
ψ	Slope of chord of internal circular arc contour

Subscripts:

c	Chamber condition
e	Exit conditions or condition at lip of shroud
ei	Condition at end of internal expansion

FORTRAN SYMBOLS

CFO	$C_{F\ opt}$	Optimum thrust coefficient
DELTA	δ	Angle between plug axis and sonic line on pure external expansion plug nozzle
G	g	Constant of proportionality in Newton's second law
GAMA	γ	Ratio of specific heats
GAM(I)	γ	Ratio of specific heats in thermodynamic table
HT	h_t/R_e	Ratio of throat gap to the radius at the shroud on pure external plug nozzle
HM(I)	M	Mach number in thermodynamic table
NT		Number of thermodynamic data
PAPC	P_a/P_c	Ratio of atmospheric pressure to chamber pressure
PXPC	P_x/P_c	Ratio of pressure at point x to chamber pressure
PEIPC	P_{ei}/P_c	Ratio of pressure at end of internal expansion to chamber pressure
PHT	ϕ_t	Angle of sonic surface to plug axis
R	R	Gas constant
RM	M_e	Exit Mach number
RMEI	M_{ei}	Mach number at end of internal expansion
RRRE	R_r/R_e	Radius of internal circular arc contour to shroud radius

FORTRAN Symbols (Cont.)

RXRE	R_x/R_e	Ratio of radius of point x to radius of shroud
SUMCG	C_{Fvacx}	Cumulative vacuum thrust coefficient
SUMIM	I_s	Cumulative specific impulse
SUMVA	I_{svac}	Cumulative vacuum specific impulse
TE	T_e	Exit temperature
VE	ν_e	Exit Prandtl-Meyer turning angle
XP	ε	Expansion ratio
XM	M_x	Mach number at the contour
XXRE	X_x/R_e	Ratio of x co-ordinate of point x to radius of shroud

TABLE OF CONTENTS

Page No.:

INTRODUCTION 1

DESIGN THEORY 2

Design of External Expansion Plug Nozzles 2

Design of Internal-External Expansion Plug Nozzles 10

THE FORTRAN PROGRAMS 17

Outline of External Expansion Plug Nozzles Design Program 17

Outline of Internal-External Plug Nozzles Design Program 18

Flow Chart of External Expansion Plug Nozzles Design Program 26

Flow Chart of Internal-External Expansion Plug Nozzles Design Program 27

REFERENCES 29

APPENDIX 30

FORTRAN Listing of External Expansion Plug Nozzles Design Program 31

FORTRAN Listing of Internal-External Plug Nozzles Design Program 34

LIST OF FIGURES

Figure No.:		Page No.:
1	External Expansion Plug Nozzle - Throat Configuration	4
2	External Expansion Plug Nozzle	6
3	Internal Portion of Internal-External Expansion Plug Nozzle	11
4	Internal-External Expansion Plug Nozzle	12
5	External Expansion Plug Nozzles Contour	19
6	Vacuum Thrust Coefficient Distribution Along the Axis of External Expansion Plug Nozzles	20
7	Vacuum Specific Impulse Distribution Along the Axis of External Expansion Plug Nozzles	21
8	Internal-External Expansion Plug Nozzles Contour	23
9	Vacuum Thrust Coefficient Distribution Along the Axis of Internal-External Expansion Plug Nozzles	24
10	Vacuum Specific Impulse Distribution Along the Axis of Internal-External Expansion Plug Nozzles	25

INTRODUCTION

The purpose of this report is to describe two FORTRAN computer programs for the design of plug nozzle contours. The theoretical method is based on the procedures described in Reference 1.

The programs described in this report are simple and provide a scheme for the design of a plug nozzle contour; however, this method becomes inaccurate as the axis of symmetry is approached. The ratio of specific heats in this program may be input either as a constant value or as a function of Mach number. The thrust coefficient, specific impulse, and dimensionless contour co-ordinates are computed at small increments along the axis of symmetry.

A complete description of the FORTRAN computer programs (including a derivation of the formula) is given in this report.

DESIGN OF EXTERNAL EXPANSION PLUG NOZZLES

In one-dimensional isentropic supersonic flow, an area ratio based on throat area can be written as follows:

$$\frac{A}{A^*} = \epsilon = \frac{1}{M}\left[\left(\frac{2}{\gamma+1}\right)\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{1}$$

where ϵ is defined as an expansion ratio. By rearranging equation (1), a function can be obtained to calculate exit Mach number:

$$f(M_e) = M_e\epsilon - \left[\frac{2+(\gamma-1)M_e^2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \quad . \tag{2}$$

Expanding this function in a Taylor's series:

$$f(M_e+\Delta M) = f(M_e) + f'(M_e)\Delta M + f''(M_e)\frac{\Delta M^2}{2!} + \text{---------} + f^n(M_e)\frac{\Delta M^n}{n!} + \text{--------} \tag{3}$$

where:

$$f'(M_e) = \epsilon - \left[\frac{2+(\gamma-1)M_e^2}{\gamma+1}\right]^{\frac{3-\gamma}{2(\gamma-1)}} \quad .$$

Truncate equation (3) at the first two terms, and assume a value M_{est} for M_e, and solve for a ΔM:

$$\Delta M_0 = -\frac{f(M_{est_0})}{f'(M_{est_0})} \quad . \tag{4}$$

A new approximation for M_e is:

$$M_{est_1} = M_{est_0} + \Delta M_0 \quad . \tag{5}$$

By carrying on this process until ΔM is within the desired limit, the exit Mach number can be obtained.

From the Prandtl-Meyer relation, a total flow turning angle can be calculated by using the following equation:

$$\nu_e = \left(\frac{\gamma+1}{\gamma-1}\right)^{\frac{1}{2}} \tan^{-1}\left[\frac{\gamma-1}{\gamma+1}\left(M_e^{2}-1\right)\right]^{\frac{1}{2}} - \tan^{-1}\left(M_e^{2}-1\right)^{\frac{1}{2}} \quad . \tag{6}$$

From the geometry of Figure 1, the following relations of throat gap can be obtained:

$$a = h_t \cos\delta \tag{7}$$

$$b = h_t \sin\delta \tag{8}$$

$$R_t = R_e - h_t \sin\delta \tag{9}$$

$$A_t = \pi(R_e - R_t)\left[a^2 + (R_e - R_t)^2\right]^{\frac{1}{2}} \tag{10}$$

$$= \pi h_t\,(2R_e - h_t \sin\delta)$$

or

$$\frac{\pi R_e^{2}}{\epsilon} = \pi h_t\,(2R_e - h_t \sin\delta) \quad . \tag{11}$$

Solving the dimensionless parameter, h_t/R_e, in the equation (11),

$$\frac{h_t}{R_e} = \frac{\epsilon - \left[\epsilon\,(\epsilon - \sin\delta)\right]^{\frac{1}{2}}}{\epsilon \sin\delta} \quad . \tag{12}$$

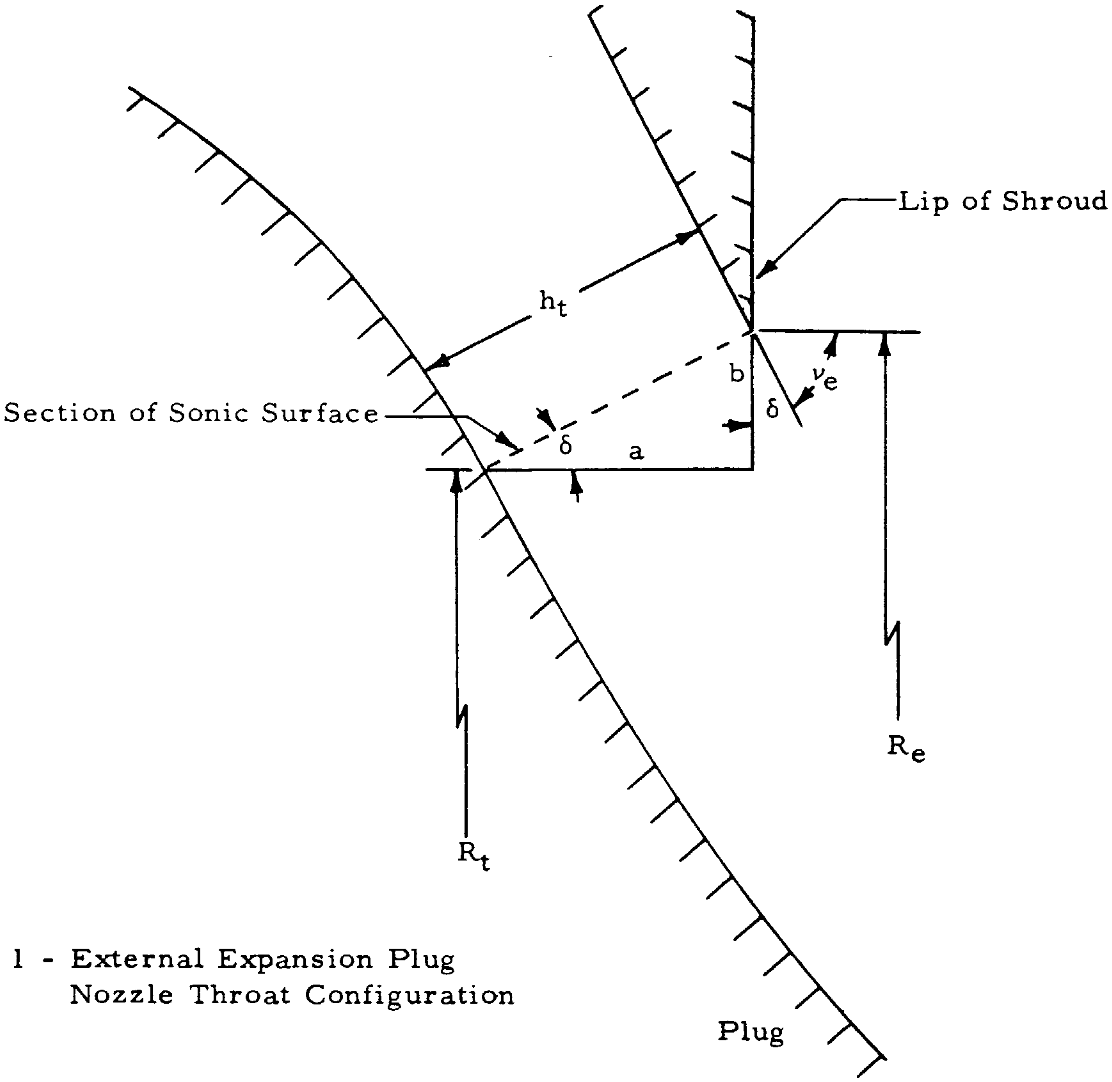


Figure 1 - External Expansion Plug Nozzle Throat Configuration

The optimum thrust coefficient, $C_{F_{opt}}$, can be calculated from the following equation.

$$C_{F_{opt}} = \frac{m V_e}{P_e A_t} = \frac{(\rho_t A_t V_t) V_e}{P_e A_t} = \frac{\rho_t V_t V_e}{P_e} = \frac{\rho_t V_t^2 M_e^*}{P_e} \quad . \tag{13}$$

By the definition of the velocity of sound in a perfect gas,

$$V_t = \frac{\gamma P_t}{\rho_t} \quad . \tag{14}$$

Equation (13) can be reduced to:

$$\begin{aligned} C_{F_{opt}} &= \gamma M_e^* \left(\frac{P_t}{P_e}\right) \\ &= \gamma M_e \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \left(1 + \frac{\gamma-1}{2} M_e\right)^{-\frac{1}{2}} \quad . \end{aligned} \tag{15}$$

The following procedure of calculation is for determining the plug contour. The Mach number on the plug surface is increased from $M_x = 1$ at the throat to $M_x = M_e$ at the tip by regular increments M_{in}.

$$M_{in} = \frac{M_e - 1}{N} \quad . \tag{16}$$

$$M_x = 1 + x M_{in} \quad . \tag{17}$$

The area of the revolved expansion wave is given by:

$$A_x = \pi (R_e - R_x) \left[X_x^2 + (R_e - R_x)^2\right]^{\frac{1}{2}} \quad . \tag{18}$$

From the geometry of Figure 2:

$$\tan \phi_x = \frac{R_e - R_x}{X_x} \quad . \tag{19}$$

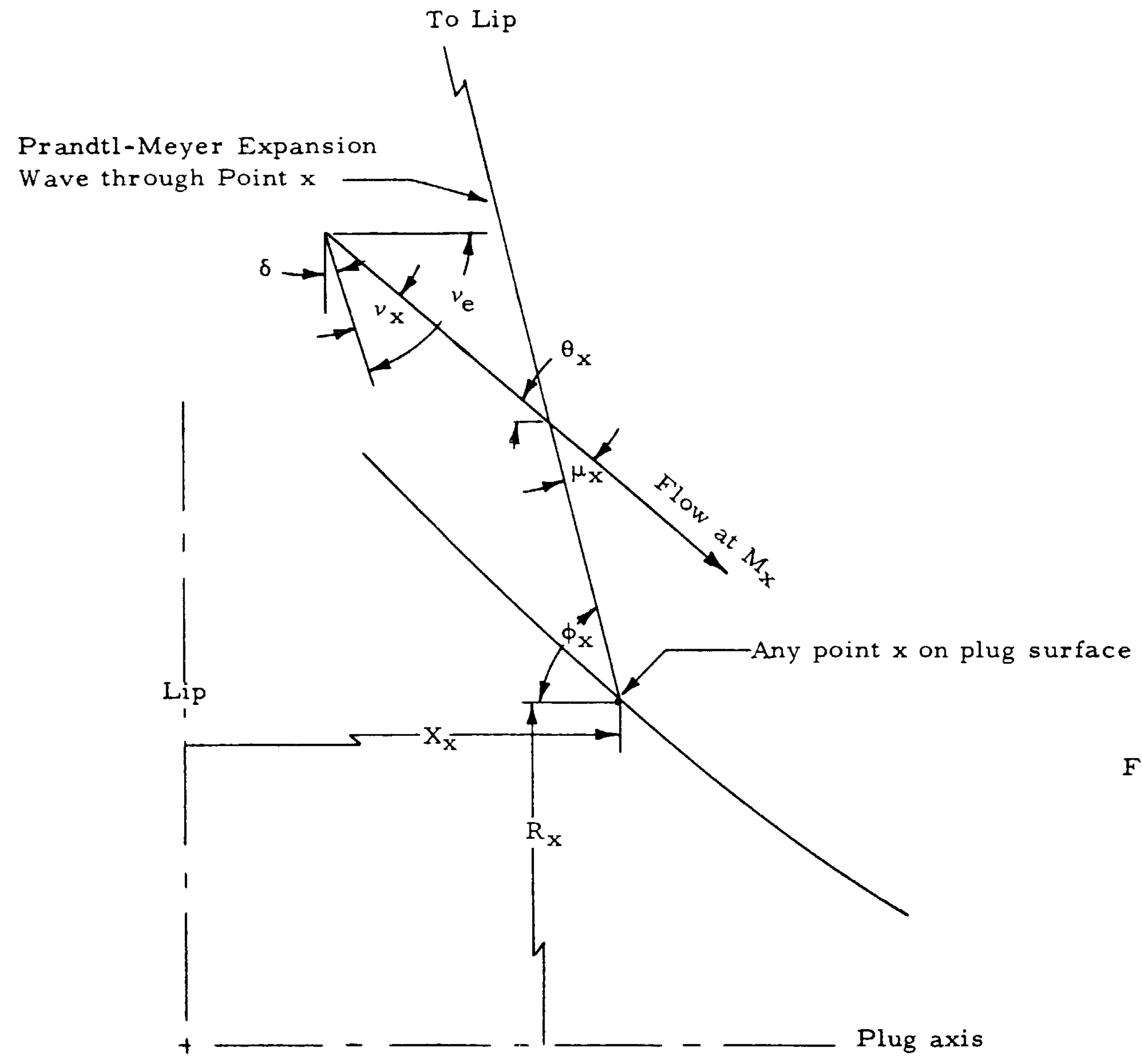


Figure 2 - External Expansion Plug Nozzle

Solving equations (18) and (19), one obtains:

$$A_x = \frac{\pi(R_e^2 - R_x^2)}{\sin \phi_x} \tag{20}$$

From the geometry of Figure 2,

$$\phi_x = \nu_e - \nu_x + \mu_x \tag{21}$$

Substitute equation (21) into (20),

$$A_x = \frac{\pi(R_e^2 - R_x^2)}{\sin(\nu_e - \nu_x + \mu_x)} \tag{22}$$

The mass flow through the revolved expansion wave is:

$$\dot{m}_x = \rho_x A_x V_x \sin \mu_x \tag{23}$$

The mass flow through the throat is:

$$\dot{m}_t = \rho_t A_t V_t \tag{24}$$

The mass flow through these two sections should be equal; therefore, A_x can be determined.

$$A_x = \frac{\dfrac{\rho_t}{\rho_e} A_t}{\dfrac{\rho_x}{\rho_e} \dfrac{V_x}{V_t} \sin \mu_x} \tag{25}$$

Equations (20) and (25) are then solved for R_x:

$$\frac{R_x}{R_e} = \left\{ 1 - \frac{\left[\left(\frac{2}{\gamma+1}\right)\left(1 + \frac{\gamma-1}{2} M_x^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} \sin(\nu_e - \nu_x + \mu_x)}{\epsilon} \right\}^{\frac{1}{2}} \tag{26}$$

Once R_x value is determined, X_x can be calculated by using equation (19).

The pressure ratio at point X can be calculated by using the following relationship:

$$\frac{P_X}{P_e} = \left(1 + \frac{\gamma - 1}{2} M_X^2\right)^{-\frac{\gamma}{\gamma-1}} \quad . \tag{27}$$

The cumulative thrust is made up of the momentum flux and the pressure thrust at the throat surface plus the pressure integral down the plug to the point in question.

$$F_X = \dot{m}_t V_t \sin\delta + (P_t - P_a) A_t \sin\delta + \int (P_X - P_a)\, dA \quad . \tag{28}$$

The corresponding specific impulse is:

$$I_s = \frac{\dot{m}_t V_t \sin\delta}{\dot{m}_t} + \frac{(P_t - P_a) A_t \sin\delta}{\rho_t A_t V_t} + \int \frac{(P_X - P_a)}{\rho_t A_t V_t} \tag{29}$$

$$= V_t \sin\delta + \frac{V_t \sin\delta (P_t - P_a)}{\rho_t \frac{\gamma P_t}{\rho_t}} + \frac{P_e V_t}{\rho_t \frac{\gamma P_t}{\rho_t}} \int \frac{(P_X - P_a)}{P_e} \frac{dA}{A_t} \tag{30}$$

$$= V_t \sin\delta + \frac{V_t \sin\delta}{\gamma} \left[1 - \left(\frac{P_a}{P_e}\right)\left(\frac{P_e}{P_t}\right)\right] + \frac{V_t}{\gamma}\left(\frac{P_e}{P_t}\right) \int \frac{(P_X - P_a)}{P_e} \frac{dA}{A_t} \quad .$$

Using isentropic relations and writing the last term in finite difference form,

$$I_s = V_t \sin\delta \left\{1 + \frac{1}{\gamma}\left[1 - \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{P_a}{P_e}\right)\right]\right\}$$

$$+ \frac{V_t}{\gamma}\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}} \sum_{n=1}^{N} \frac{\epsilon}{2}\left[\left(\frac{P_X - P_a}{P_e}\right)_{n-1} + \left(\frac{P_X - P_a}{P_e}\right)_n\right]\left[\left(\frac{R_X}{R_e}\right)_{n-1}^2 - \left(\frac{R_X}{R_e}\right)_n^2\right] \quad . \tag{31}$$

The vacuum thrust coefficient is:

$$\begin{aligned}
C_{F_{vac}} &= \frac{M_t V_t \sin\delta}{P_e A_t} + \frac{P_t A_t \sin\delta}{P_e A_t} + \int \frac{P_x}{P_e}\,\frac{dA}{A_t} \\
&= \left(\frac{\rho_t A_t V_t^2 \sin\delta}{P_e A_t}\right) + \left(\frac{P_t}{P_e}\right)\sin\delta + \int \frac{P_x}{P_e}\,\frac{dA}{A_x} \\
&= \left(\frac{\rho_t \, \frac{\gamma P_t}{\rho_t} \sin\delta}{P_e}\right) + \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} + \int \frac{P_x}{P_e}\,\frac{dA}{A_x} \\
&= \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}(\gamma+1)\sin\delta + \sum_{n=1}^{N}\left[\left(\frac{P_x}{P_e}\right)_{n-1} + \left(\frac{P_x}{P_e}\right)_n\right]\left[\left(\frac{R_x}{R_e}\right)_{n-1}^2 - \left(\frac{R_x}{R_e}\right)_n^2\right] .
\end{aligned} \tag{32}$$

DESIGN OF INTERNAL-EXTERNAL PLUG NOZZLES

The internal expansion is assumed to occur as a simple wave expansion around an initial circular arc contour. The external expansion is assumed to occur as before, namely, a center simple wave or Prandtl-Meyer expansion about the lip of the shroud.

Equations (2), (3), (4), (5), and (6) are used to calculate the exit Mach number and the total flow turning angle. If the pressure ratio at the end of internal expansion, P_{ei}/P_e is specified, the Mach number at the end of internal expansion can be determined by using the following equation:

$$M_{ei} = \left\{ \frac{2}{\gamma - 1} \left[\left(\frac{P_{ei}}{P_e} \right)^{-\frac{\gamma - 1}{\gamma}} - 1 \right] \right\}^{\frac{1}{2}} \quad . \tag{33}$$

The internal flow turning angle can be obtained from the Prandtl-Meyer relation:

$$\nu_{ei} = \left(\frac{\gamma + 1}{\gamma - 1} \right)^{\frac{1}{2}} \tan^{-1} \left[\frac{\gamma - 1}{\gamma + 1} (M_{ei}^2 - 1) \right]^{\frac{1}{2}} - \tan^{-1} (M_{ei}^2 - 1)^{\frac{1}{2}} \quad . \tag{34}$$

The slope of the last internal expansion wave is:

$$\phi_{ei} = \theta_{ei} + \mu_{ei} \tag{35}$$

where:

$$\theta_{ei} = \theta_t - \nu_{ei} \quad . \tag{36}$$

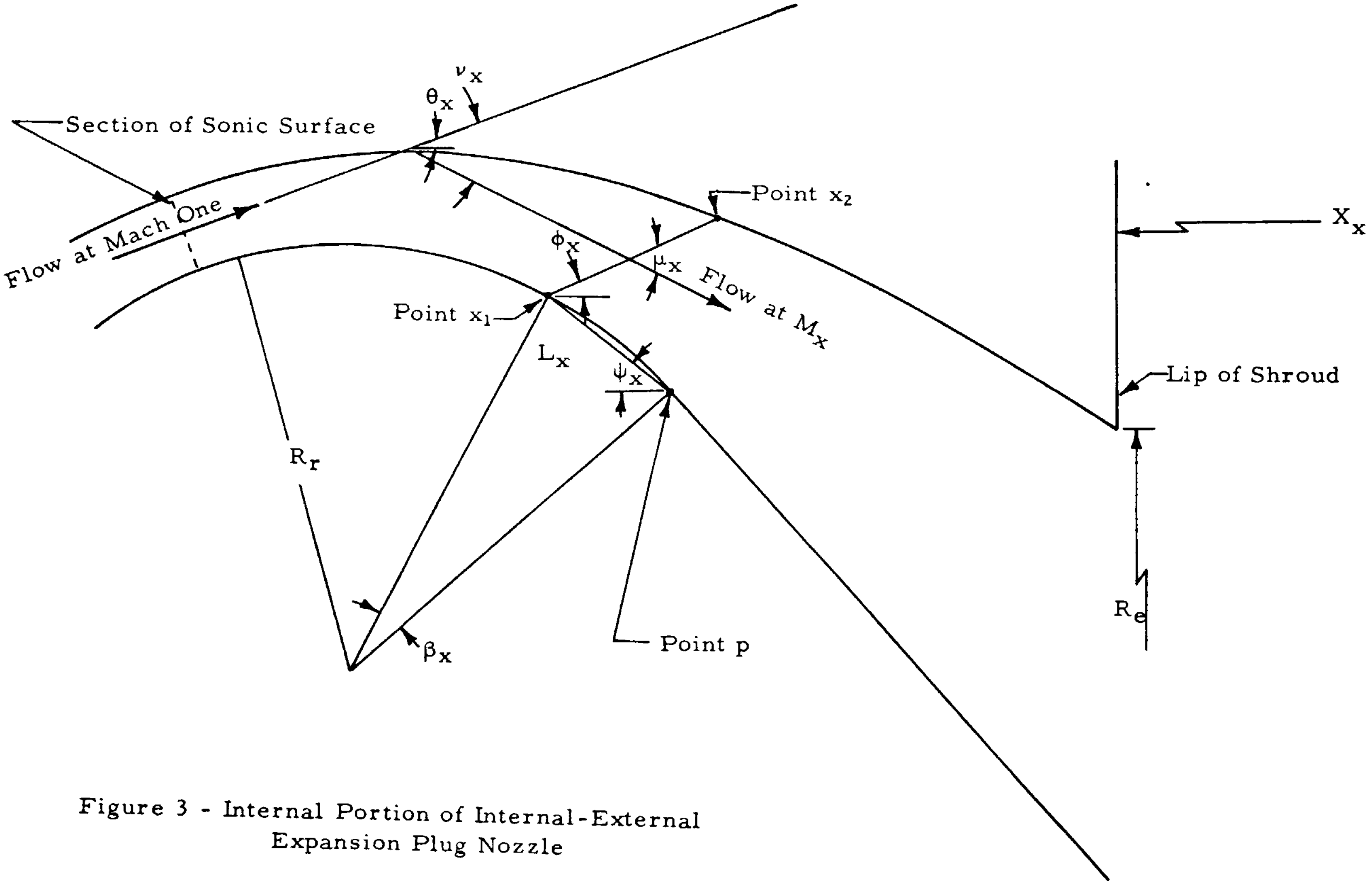


Figure 3 - Internal Portion of Internal-External Expansion Plug Nozzle

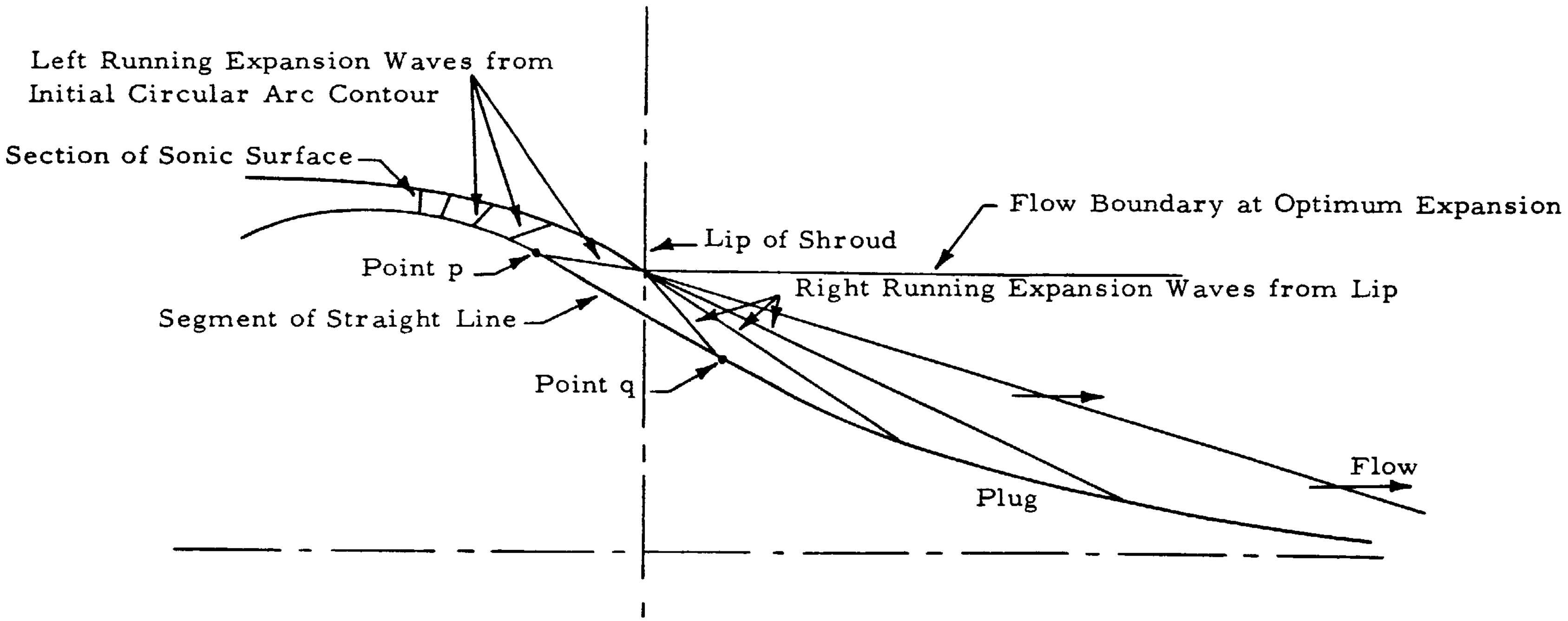


Figure 4 - Internal-External Expansion Plug Nozzle

Since point P, the origin of the last internal expansion wave, is located on the plug contour, equations (19) and (26) can be used to calculate its co-ordinates.

$$\frac{R_p}{R_e} = \left[1 - \frac{\left[\left(\frac{2}{\gamma+1} \right) \left(1 - \frac{\gamma-1}{2} M_{ei}^{2} \right) \right]^{\frac{\gamma+1}{2(\gamma-1)}} \sin \phi_{ei}}{\epsilon} \right] . \tag{37}$$

$$\frac{X_p}{R_e} = \frac{\frac{R_p}{R_e} - 1}{\tan \phi_{ei}} . \tag{38}$$

The first part of the contour calculation is similar to those of the previous section:

$$M_{in} = \frac{M_{ei} - 1}{N_1} . \tag{39}$$

$$M_x = 1 + x M_{in} . \tag{40}$$

The Prandtl-Meyer angle at any location is now calculated from:

$$\nu_x = \left(\frac{\gamma+1}{\gamma-1} \right)^{\frac{1}{2}} \tan^{-1} \left[\frac{\gamma-1}{\gamma+1} (M_x^2 - 1) \right]^{\frac{1}{2}} - \tan^{-1} (M_x^2 - 1)^{\frac{1}{2}} . \tag{41}$$

The flow of point P is assumed to be perpendicular to the radius of the circular arc contour. The central angle, β_x, can be obtained from:

$$\beta_x = \phi_t - 90^\circ - \nu_x + \left| \theta_e \right| . \tag{42}$$

The chord length is equal to:

$$\frac{L_x}{R_e} = 2\,\frac{R_r}{R_e}\,\sin\frac{1}{2}\,\beta_x \quad . \tag{43}$$

From the geometry of the Figure 3,

$$\psi_x = 180 - \phi_t + \nu_x - \frac{180 - \beta_x}{2} \quad . \tag{44}$$

The co-ordinates of the point X_1 can be determined from the following equations:

$$\frac{R_{x_1}}{R_e} = \frac{R_p}{R_e} + \frac{L_x}{R_e}\sin\psi_x \tag{45}$$

and

$$\frac{X_{x_1}}{R_e} = \frac{X_p}{R_e} - \frac{L_x}{R_e}\cos\psi_x \quad . \tag{46}$$

The derivation of the calculation of point X_2 is similar to that used in equation (26).

$$\frac{R_{x_2}}{R_e} = \left\{\left(\frac{R_{x_1}}{R}\right)^2 + \left[\left(\frac{2}{\gamma+1}\right)\left(1 + \frac{\gamma-1}{2}M_x^{2}\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}\sin\phi_x\right\}^{\frac{1}{2}} \quad . \tag{47}$$

$$\frac{X_{x_2}}{R_e} = \frac{X_{x_1}}{R_e} + \frac{\frac{R_{x_2}}{R_e} - \frac{R_{x_1}}{R_e}}{\tan\phi_x} \tag{48}$$

where:

$$\phi_x = 2\nu_{e_i} - \nu_e - \nu_x + \mu_x \quad . \tag{49}$$

Equation (27) can be used to calculate the pressure ratio at points X_1 and X_2. When M_x has been incremented from $M_x = 1$ to $M_x = M_{ei}$, the design of the internal portion of the nozzle is complete.

The design of the external portion can be carried on by using the following relations:

(1) The last expansion wave from the initial circular arc contour at point P is a member of a family of left running waves and intersects the lip of the shroud which is shown in Figure 4.

(2) The remaining expansion to the exit Mach number occurs about the lip of the shroud and is made up of a family of right-running expansion waves.

(3) Flow properties on the first of the right-running wave are equal to those on the last left-running wave.

(4) The external contour is determined in the same manner as for a pure external expansion nozzle.

The cumulative thrust is computed by considering the momentum flux and pressure thrust at the first right-running external expansion wave and the pressure integral on the remainder of the plug.

$$F_x = \dot{m}_t V_g \cos\theta_g + (P_g - P_a) A_g \sin\phi_g + \int (P_x - P_a)\, dA \ . \tag{50}$$

$$\begin{aligned} I_{sp} &= \frac{F_x}{\dot{m}_t} \\ &= V_g \cos\theta_g + \frac{(P_g - P_a)}{\rho_g V_g} \sin\phi_g + \int \frac{(P_x - P_a)}{\rho_t A_t V_t}\, dA \\ &= V_g \left\{ \cos\theta_g + \frac{1}{\gamma}\left[1 - \left(\frac{P_a}{P_e}\right)\left(1 + \frac{\gamma-1}{2} M_g^{\,2}\right)^{\frac{\gamma}{\gamma-1}} \frac{\sin\phi_g}{M_g^{\,2}}\right] \right. \\ &\quad + \frac{V_t}{\gamma}\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}} \sum_{n=1}^{N_2} \frac{\epsilon}{2}\left[\left(\frac{P_x - P_a}{P_e}\right) + \left(\frac{P_x - P_a}{P_e}\right)_n\right]\left[\left(\frac{R_x}{R_e}\right)^2_{n-1} - \left(\frac{R_x}{R_e}\right)^2_n\right] \ , \end{aligned} \tag{51}$$

where:

$$V_q = \left[KR \; \frac{T_e \left(1 + \frac{\gamma - 1}{2} M_e^2\right)}{1 + \frac{\gamma - 1}{2} M_q^2} \right]^{\frac{1}{2}}$$

and

$$V_t = \left[KR \; \frac{T_e \left(1 + \frac{\gamma - 1}{2} M_e^2\right)}{\frac{\gamma + 1}{2}} \right]^{\frac{1}{2}} \quad .$$

The cumulative vacuum thrust coefficient can be calculated as follows:

$$\begin{aligned} C_{F_{vac_x}} &= \frac{\gamma P_t V_q \cos\theta_q}{P_e V_t} + \epsilon \frac{P_q}{P_e}\left[1 - \left(\frac{R_q}{R_e}\right)^2\right] + \int \frac{P_x \, dA}{P_e A_t} \\ &= \gamma \left(\frac{P_t}{P_e}\right)\left(\frac{V_q}{V_t}\right) \cos\theta_q + \epsilon \frac{P_q}{P_e}\left[1 - \left(\frac{R_q}{R_e}\right)^2\right] \\ &\quad + \sum_{n=1}^{N_2} \frac{\epsilon}{2}\left[\left(\frac{P_x}{P_e}\right)_{n-1} + \left(\frac{P_x}{P_e}\right)_n\right]\left[\left(\frac{R_x}{R_e}\right)_{n-1}^2 - \left(\frac{R_x}{R_e}\right)_n^2\right] \quad . \end{aligned} \tag{52}$$

THE FORTRAN PROGRAMS

Outline of External Expansion Plug Nozzle Design

INPUT:

(1) Estimated exit Mach number (which can be obtained from isentropic flow tables based on the expansion ratio and the ratio of specific heats)

(2) Expansion ratio

(3) Number of contour points

(4) Gas constant

(5) Exit temperature

(6) Atmosphere pressure ratio

(7) Constant of proportionality in Newton's second law

(8) Ratio of specific heats (constant or variable)

OUTPUT:

(1) Angle between plug axis and sonic line

(2) Width of throat gap

(3) Optimum thrust coefficient

(4) Mach number distribution

(5) Co-ordinates of plug contour

(6) Pressure ratio at each point

(7) Cumulative vacuum thrust coefficient

(8) Cumulative specific impulse

(9) Cumulative vacuum specific impulse

This program has been used to compute a few examples. The results of design nozzle contours are shown in Figure 5. The vacuum thrust coefficient and vacuum specific impulse distributions along the plug axis are shown in Figure 6 and Figure 7 respectively.

Outline of Internal-External Expansion Plug Nozzle Design

INPUT:

(1) Number of internal contour points and external contour points

(2) Pressure ratio at end of internal expansion

(3) Expansion ratio

(4) Radius of internal circular arc contour

(5) Estimated Mach number

(6) Angle between plug axis and Prandtl-Meyer expansion wave at threat

(7) Gas constant

(8) Exit temperature

(9) Atmosphere pressure ratio

(10) Constant of proportionality in Newton's second law

(11) Ratio of specific heats (constant or variable)

OUTPUT:

(1) Mach number distribution

(2) Co-ordinates of nozzle contour

(3) Pressure ratio at each point

(4) Cumulative vacuum thrust coefficient

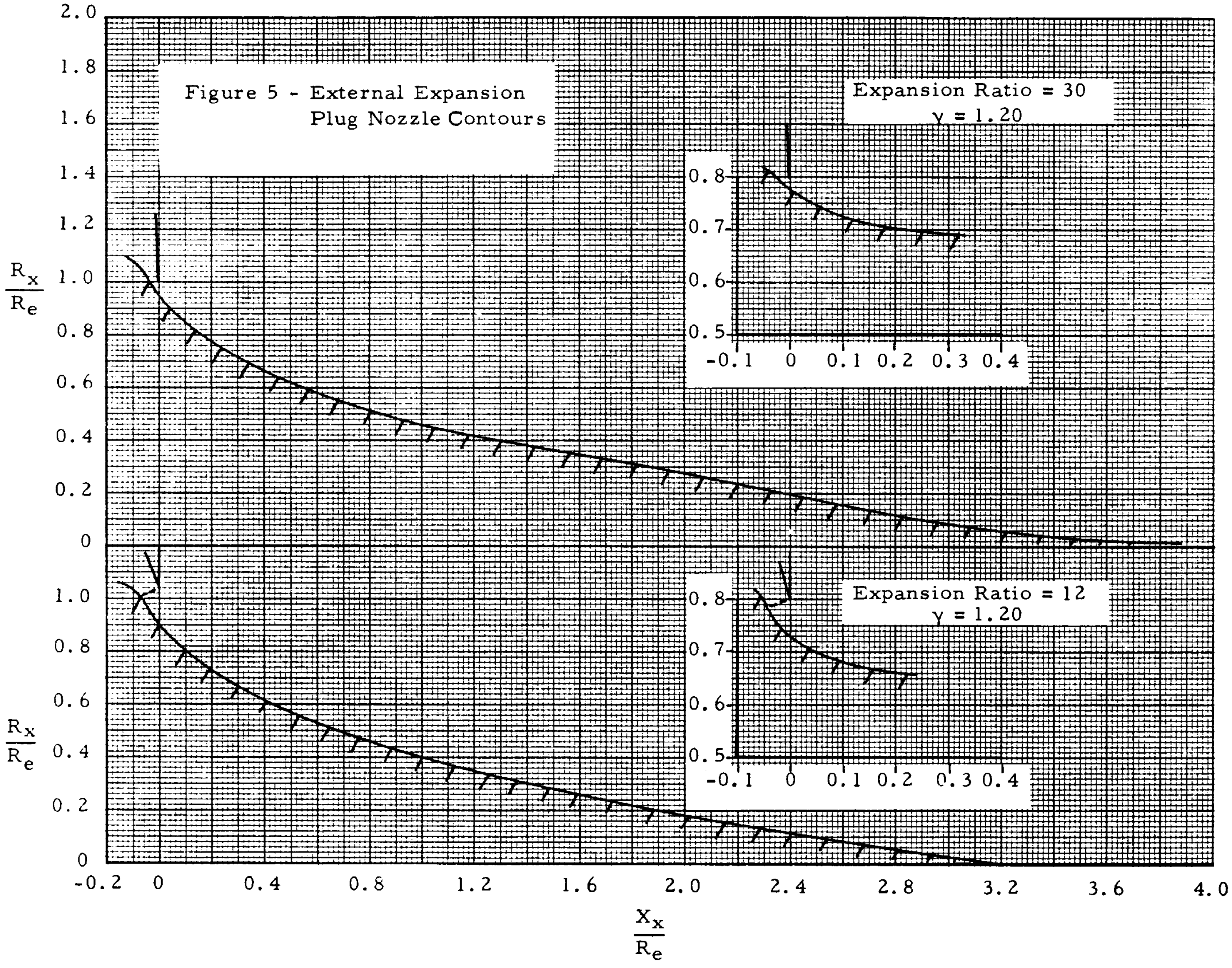
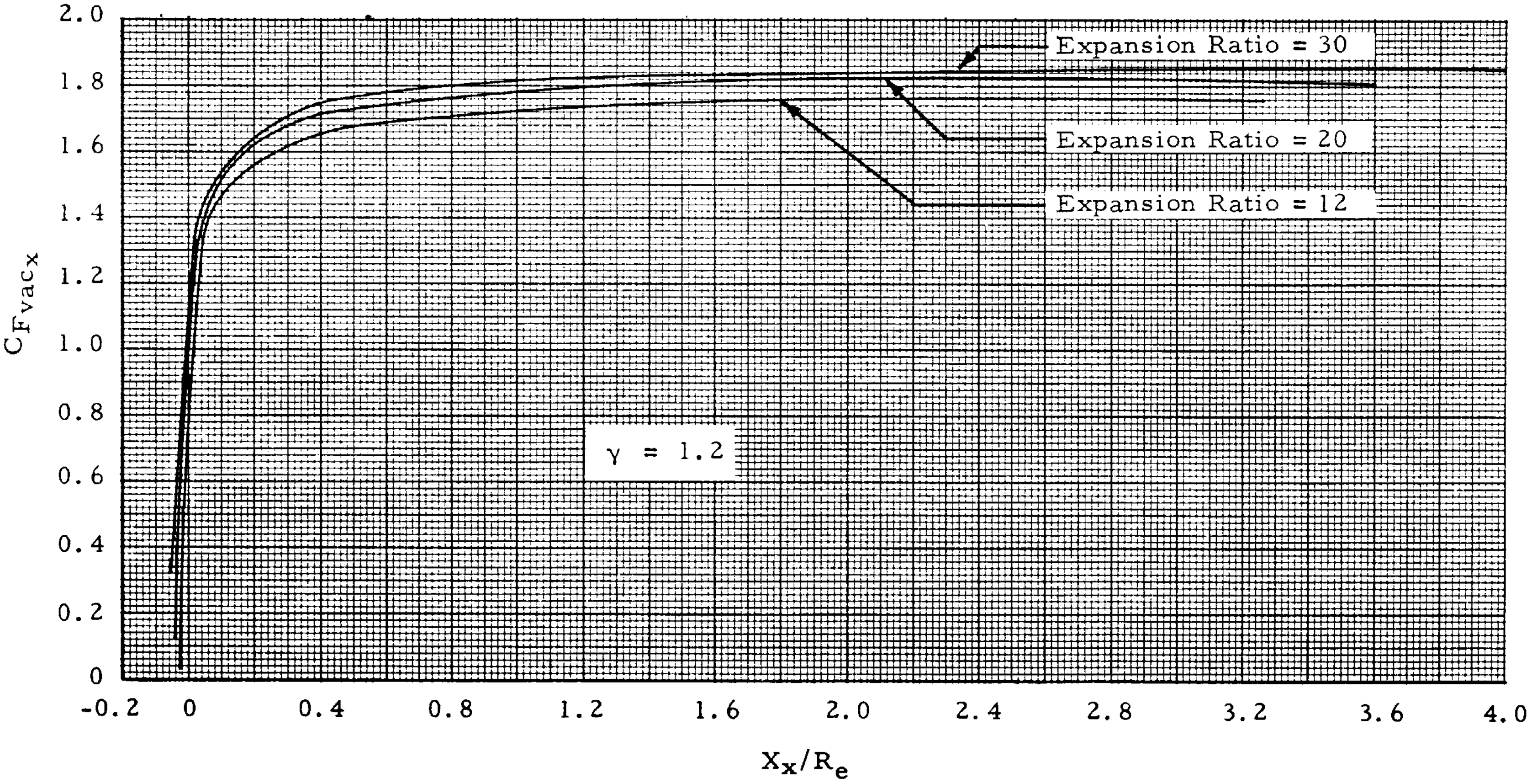


Figure 5 - External Expansion Plug Nozzle Contours

Figure 6 - Vacuum Thrust Coefficient Distribution Along the Axis of External Expansion Plug Nozzles



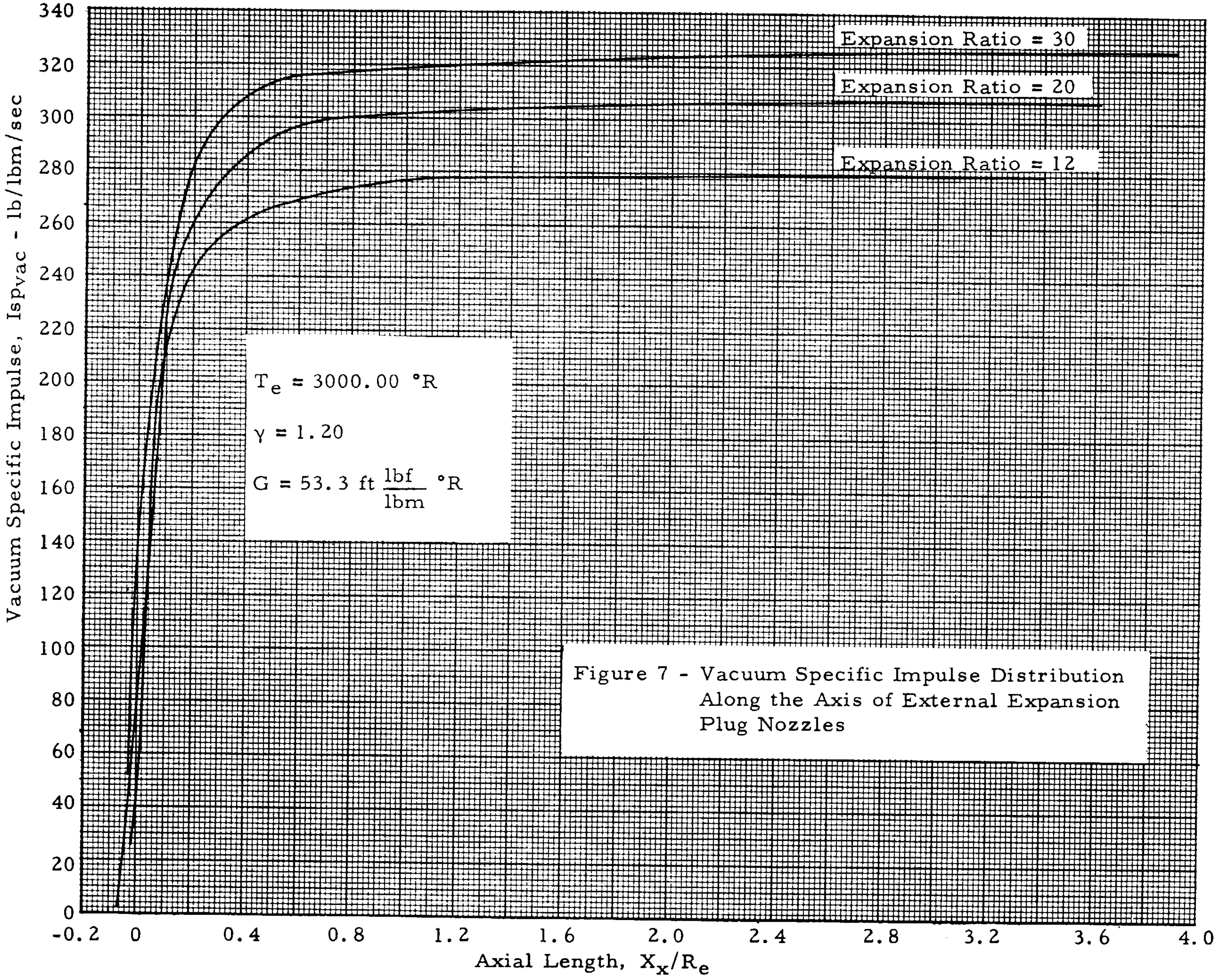


Figure 7 - Vacuum Specific Impulse Distribution Along the Axis of External Expansion Plug Nozzles

(5) Cumulative specific impulse

(6) Cumulative vacuum specific impulse

This program has been used to compute a few examples. The results of design nozzle contour are shown in Figure 8. The vacuum thrust coefficient and vacuum specific impulse distributions along the plug axis are shown in Figures 9 and 10 respectively.

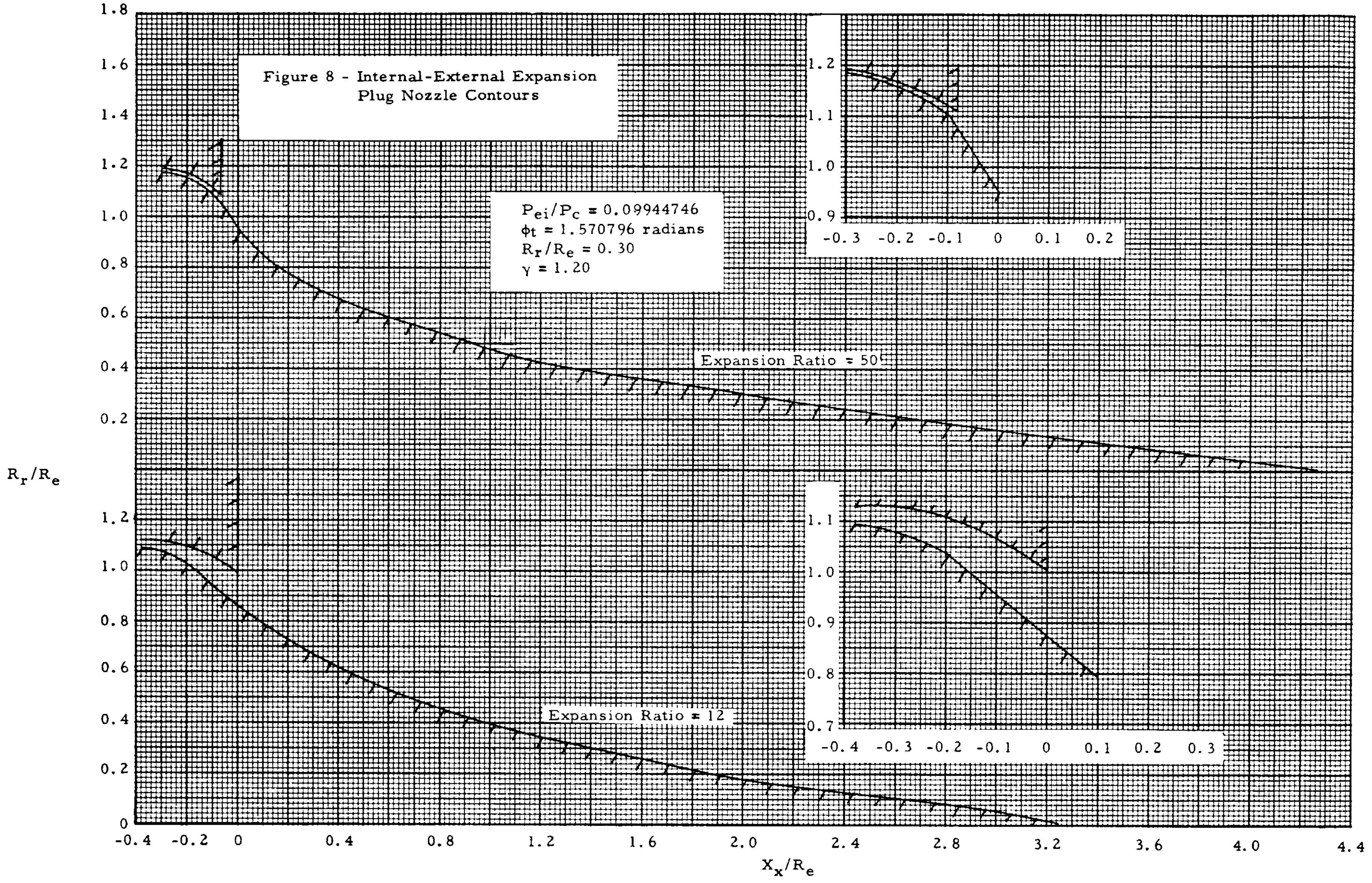


Figure 8 - Internal-External Expansion Plug Nozzle Contours

Figure 9 - Vacuum Thrust Coefficient Distribution Along the Axis of Internal-External Expansion Plug Nozzles

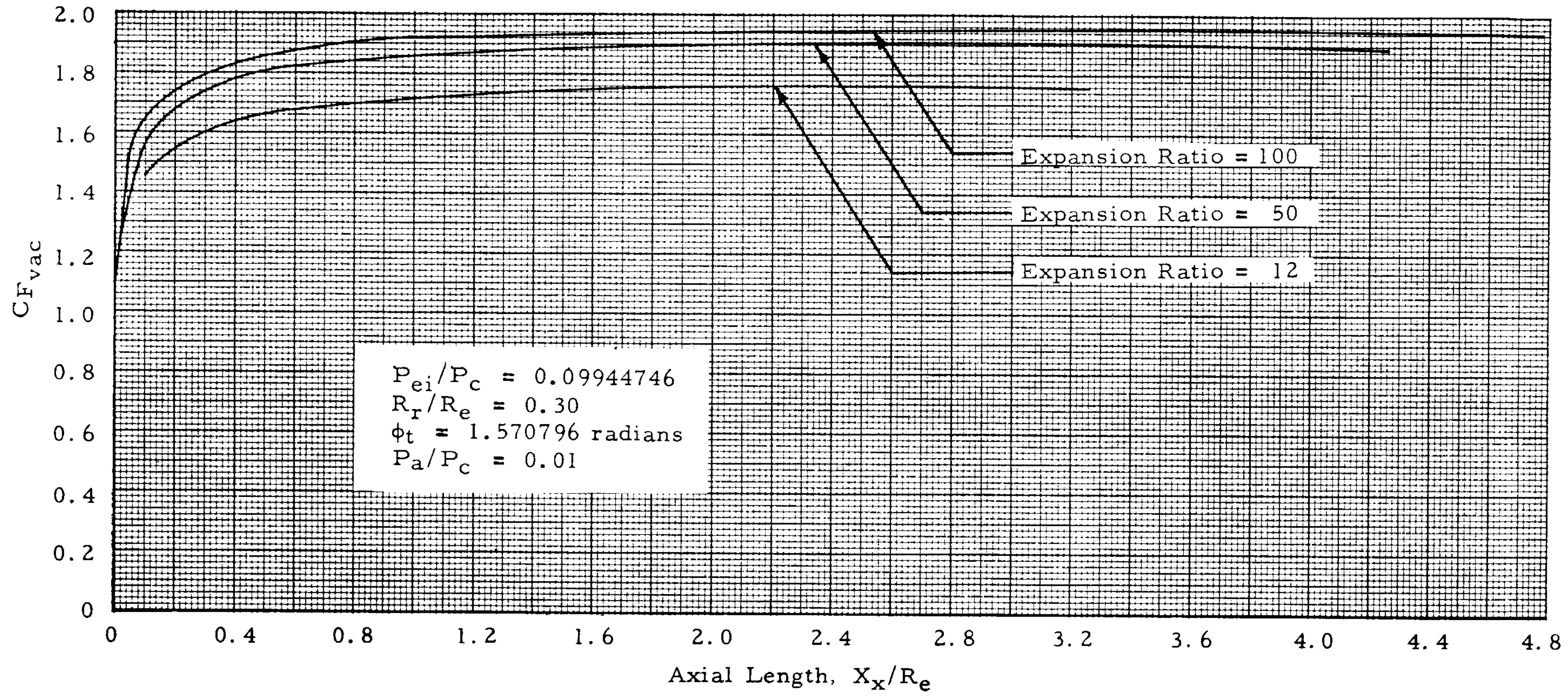
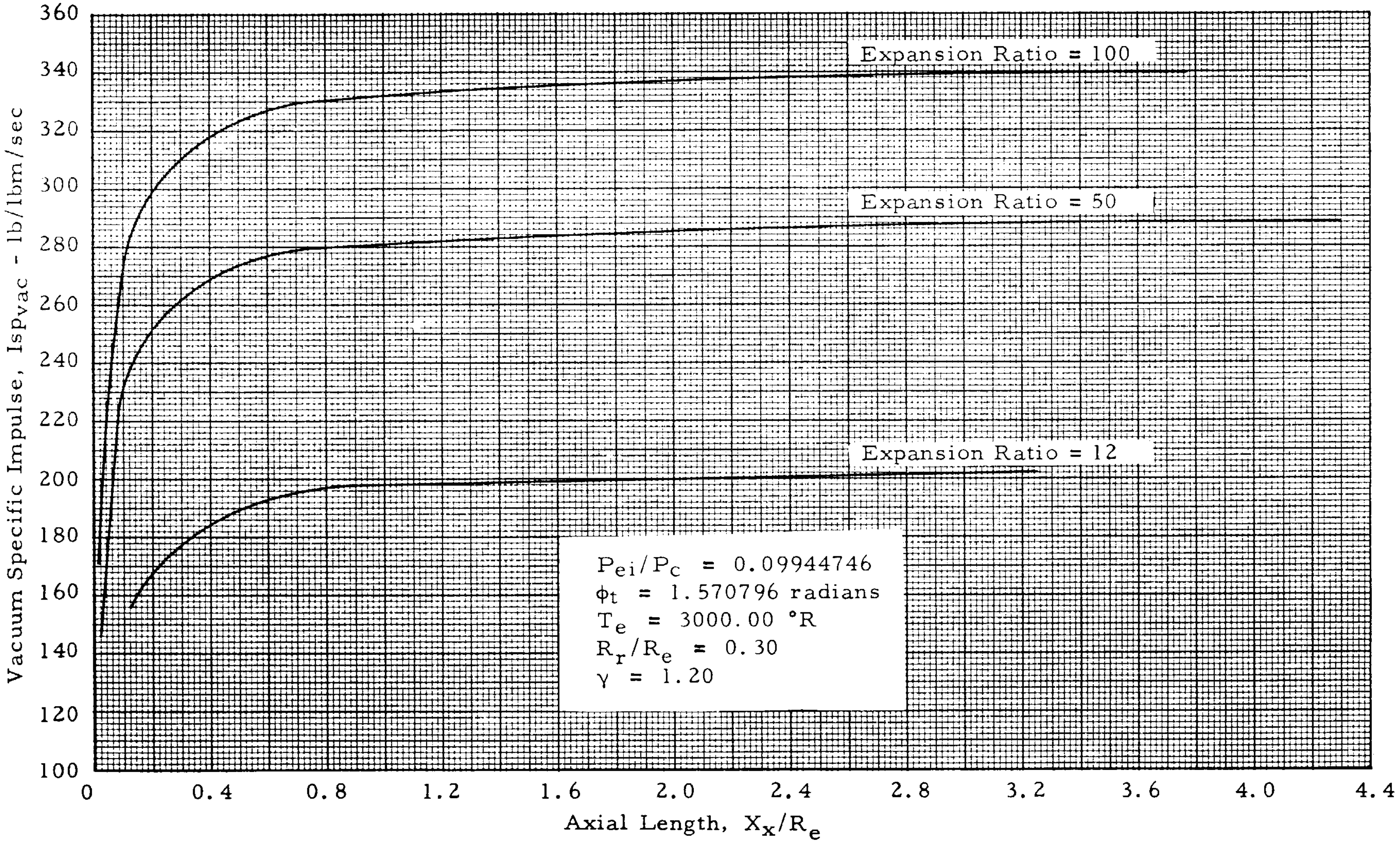
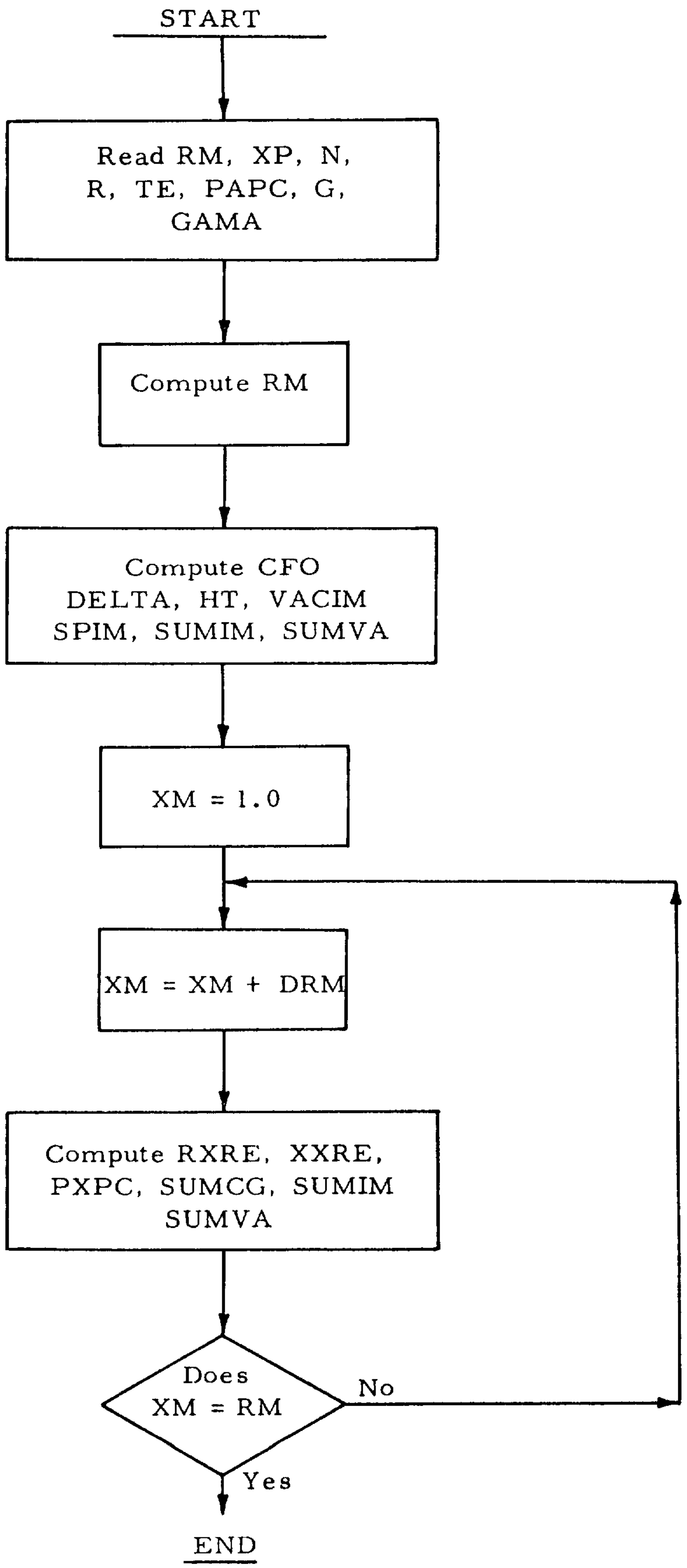


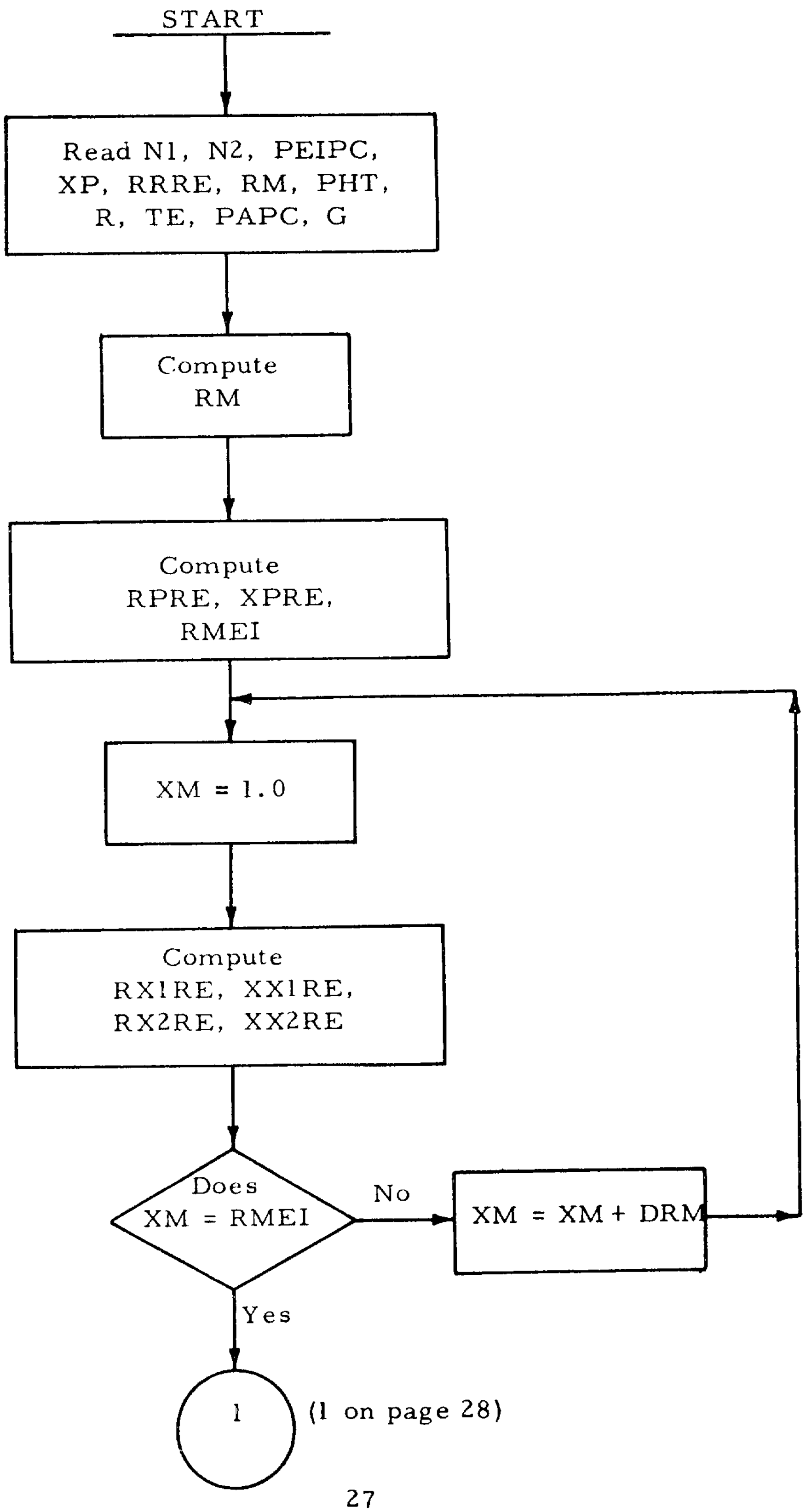
Figure 10 - Vacuum Specific Impulse Distribution Along the Axis of Internal-External Expansion Plug Nozzles



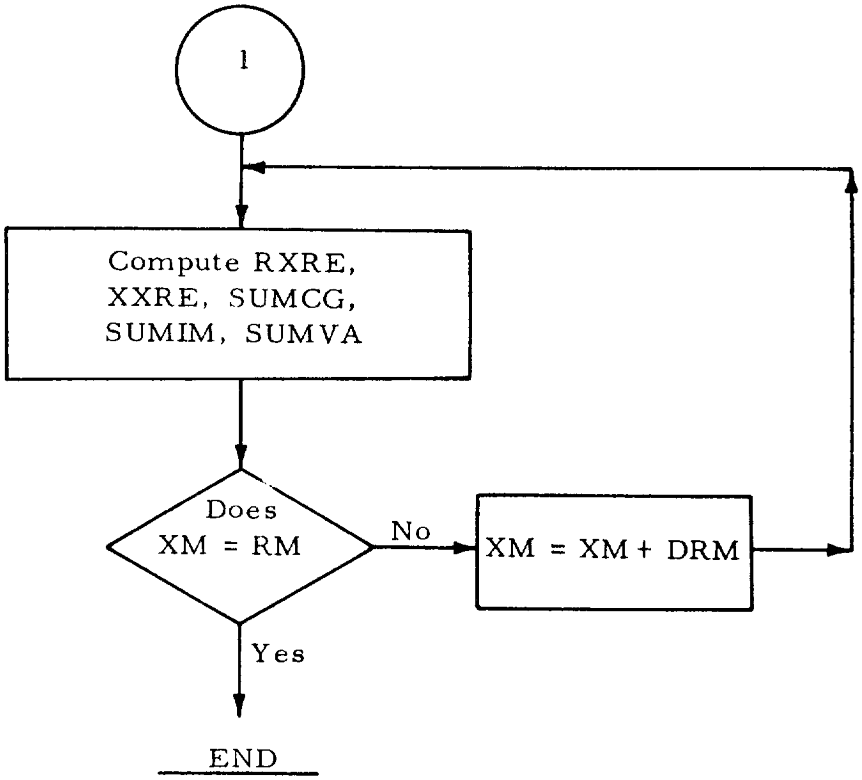
FLOW CHART OF EXTERNAL EXPANSION PLUG NOZZLES DESIGN



FLOW CHART OF INTERNAL-EXTERNAL EXPANSION PLUG NOZZLES DESIGN



(1 on page 28)


1
Compute RXRE, XXRE, SUMCG, SUMIM, SUMVA
Does XM = RM
No
XM = XM + DRM
Yes
END

REFERENCES

1. T. L. Deyound, "A Simplified Method for Plug Nozzle Design", Technical Memorandum No. 140, July, 1960

2. Ascher H. Shapiro, *The Dynamics and Thermodynamics of Compressible Fluid Flow*, The Ronald Press Company, Vol. 1

3. K. Berman and F. W. Crimp, Jr., "Performance of Plug-Type Rocket Exhaust Nozzles", ARS Solid Propellant Rocket Research Conference, Princeton, New Jersey, January 28-29, 1960

APPENDIX

```
C       DESIGN OF EXTERNAL EXPANSION PLUG NOZZLES
C       INPUT GAS#&1.0 WHEN DEALING WITH IDEAL GAS
C       INPUT GAS#-1.0 WHEN DEALING WITH REAL GAS
        DIMENSION HM%30□,GAM%30□
    101 READ1,RM
        READ1,GAS
      1 FORMAT%F4.0□
        PRINT47,RM
     47 FORMAT%1H1,8HESTIMATE,1X,4HMACH,1X,6HNUMBER,1H#,F10.5□
        READ2,XP,N
      2 FORMAT%F10.0,I4□
        PRINT102,XP
    102 FORMAT%1HK,9HEXPANSION,1X,5HRATIO,1X,1H#,F10.5□
        READ60,R,TE,PAPC,G
     60 FORMAT%4F10.0□
        PRINT51,R
     51 FORMAT%1HK,3HGAS,1X,8HCONSTANT,1X,1H#,E14.7□
        PRINT61,TE
     61 FORMAT%1HK,4HEXIT,1X,11HTEMPERATURE,1X,1H#,E14.7□
        PRINT53,PAPC
     53 FORMAT%1HK,5HPA/PC,1X,1H#,E14.7□
        IF%GAS□4,4,10
     10 READ3,GAMA
      3 FORMAT%F5.0□
        PRINT 45
     45 FORMAT%1HK,5HUSING,1X,5HIDEAL,1X,3HGAS□
        PRINT49,GAMA
     49 FORMAT%1HK,1X,4HGAMA,1X,1H#,F5.2□
        GO TO 9
C       REAL GAS HAS TO INPUT NT VALUES OF THERMODYNAMIC DATA
      4 READ5,NT
      5 FORMAT%I4□
        PRINT 46
     46 FORMAT%1HK,5HUSING,1X,4HREAL,1X,3HGAS□
        DO8I#1,NT
        READ6,HM%I□,GAM%I□
      6 FORMAT%2F10.7□
      8 CONTINUE
      9 CONTINUE
     13 IF%GAS□30,31,31
     30 DO34J#1,NT
        I#J
        IF%RM-HM%J□□32,33,34
     34 CONTINUE
     33 GAMA#GAM%I□
        GO TO 31
     32 GAMA#GAM%I-1□&%RM-HM%I-1□□*%GAM%I□-GAM%I-1□□/%HM%I□-HM%I-1□□
     31 FME#%2.0&%GAMA-1.0□*RM*RM□/%GAMA&1.0□
        COM#%GAMA&1.0□/%2.0*%GAMA-1.0□□
        FME#RM*XP-FME**COM
        FPM#%2.0&%GAMA-1.0□*RM*RM□/%GAMA&1.0□
        COM#%3.0-GAMA□/%2.0*%GAMA-1.0□□
```

```
      FPM#XP-RM*%FPM**COM□
      DM#-FME/FPM
      RM#RM&DM
      DM#ABSF%DM□
      IF%DM-0.00001□12,12,13
   12 CONTINUE
      A#SQRTF%%GAMA-1.0□*%RM*RM-1.0□/%GAMA&1.0□□
      B#SQRTF%%GAMA&1.0□/%GAMA-1.0□□
      C#SQRTF%RM*RM-1.0□
      C#ATANF%C□
      VE#B*ATANF%A□-C
      DELTA#1.570796-VE
      SUMCG#%%2.0/%GAMA&1.0□□**%GAMA/%GAMA-1.□□□*%GAMA&1.□*SINF%DELTA□
      HT#%XP-SQRTF%XP*%XP-SINF%DELTA□□□□/%XP*SINF%DELTA□□
      A1#%GAMA&1.0□/%2.0*%GAMA-1.0□□
      B1#SQRTF%1.0&0.5*%GAMA-1.0□*RM*RM□
      CFO#GAMA*RM*%%2.0/%GAMA&1.0□□**A1□/B1
      PCPT#%0.5*%GAMA&1.0□□**%GAMA/%GAMA-1.0□□
      VT#1.0&0.5*%GAMA-1.0□*RM*RM
      VT#GAMA*R*TE*VT/%0.5*%GAMA&1.0□□
      VT#VT*G
      VT#SQRTF%VT□
      SPIM#%1.0-PCPT*PAPC□/GAMA
      SPIM#1.0&SPIM
      SUMIM#VT*SINF%DELTA□*SPIM/G
      VACIM#1.0&1.0/GAMA
      SUMVA#VT*SINF%DELTA□*VACIM/G
      PRINT 15
   15 FORMAT%1HK,5HDELTA,9X,5HHT/RE,9X,5HCFOPT□
      PRINT 16,DELTA,HT,CFO
   16 FORMAT%1HK,3E 14.7□
      XN#N
      DRM#%RM-1.0□/XN
      XM#1.0
      K#1
      RXRE#1.0-HT*SINF%DELTA□
      XXRE#%-HT□*COSF%DELTA□
      IF%CAS□38,39,39
   38 DO 37J#1,NT
      I#J
      IF%XM-HM%J□□35,36,37
   37 CONTINUE
   36 GAMA#GAM%I□
      GO TO 39
   35 GAMA#GAM%I-1□&%XM-HM%I-1□□*%GAM%I□-GAM%I-1□□/%HM%I□-HM%I-1□□
   39 A3#%-GAMA□/%GAMA-1.0□
      PXPC#%1.0&0.5*%GAMA-1.0□*XM*XM□**A3
      PRINT17
   170FORMAT%1HK,4HMACH,10X,5HRX/RE,9X,5HXX/RE,9X,5HPX/PC,9X,5HCFVAC,
     19X,3HSP.,1X,7HIMPULSE,3X,4HVAC.,1X,7HIMPULSE□
      GO TO 22
   14 K#K&1
```

80-80 CARD TO PRINTER

```
 50 IF%GAS¤41,40,40
 41 DO44J#1,NT
    I#J
    IF%XM-HM%J¤¤42,43,44
 44 CONTINUE
 43 GAMA#GAM%I¤
    GO TO 40
 42 GAMA#GAM%I-1¤&%XM-HM%I-1¤¤*%GAM%I¤-GAM%I-1¤¤/%HM%I¤-HM%I-1¤¤
 40 A#SCRTF%%GAMA-1.0¤*%XM*XM-1.0¤/%GAMA&1.0¤¤
    B#SCRTF%%GAMA&1.0¤/%GAMA-1.0¤¤
    C#SCRTF%XM*XM-1.0¤
    C#ATANF%C¤
    VX#B*ATANF%A¤-C
    Y#1.0/XM
    UX#ATANF%Y/SQRTF%1.0-Y*Y¤¤
    A2#%GAMA&1.0¤/%2.0*%GAMA-1.¤¤
    B2#%2.0/%GAMA&1.0¤¤*%1.0&0.5*%GAMA-1.0¤*XM*XM¤
    RXRE# 1.0-%B2**A2¤*SINF%VE-VX&UX¤/XP
    RXRE#SQRTF%RXRE¤
 52 XXRE#%1.0-RXRE¤*COSF%VE-VX&UX¤/SINF%VE-VX&UX¤
    A3#%-GAMA¤/%GAMA-1.0¤
    PXPC#%1.0&0.5*%GAMA-1.0¤*XM*XM¤**A3
    SUMCG#SUMCG&0.5*XP*%PRO&PXPC¤*%RXO*RXO-RXRE*RXRE¤
    CO#PCPT*VT*XP/%G*GAMA¤
    SUMIM#SUMIM&0.5*CO*%PRO&PXPC-2.0*PAPC¤*%RXO*RXO-RXRE*RXRE¤
    SUMVA#SUMVA&0.5*CO*%PRO&PXPC¤*%RXO*RXO-RXRE*RXRE¤
 22 PRINT18,XM,RXRE,XXRE,PXPC,SUMCG,SUMIM,SUMVA
 18 FORMAT%1HK,7E14.7¤
    IF%K-N¤19,19,20
 19 XM#XM&DRM
    PRO#PXPC
    RXO#RXRE
    GO TO 14
 20 PRINT21
 21 FORMAT%1HK, 8HEXTERNAL,1X,9HEXPANSION,1X,6HNOZZLE,1X,7HCONTOUR¤
    GO TO 101
    END
```

80-80 CARD TO PRINTER

```
C       DESIGN OF INTERNAL-EXTERNAL EXPANSION PLUG NOZZLES
C       INPUT GAS#&1.0 WHEN DEALING WITH IDEAL GAS
C       INPUT GAS#-1.0 WHEN DEALING WITH REAL GAS
        DIMENSION HM%30¤,GAM%30¤
  101   READ11,N1,N2
   11   FORMAT%2I4¤
        READ1,GAS,PEIPC,XP,RRRE,RM,PHT
    1   FORMAT%6F10.0¤
        PRINT52,PEIPC
   52   FORMAT%1H1,6HPEI/PC,1X,1H#,E14.7¤
        PRINT53,XP
   53   FORMAT%1HK,9HEXPANSION,1X,5HRATIO,1X,1H#,F10.5¤
        PRINT54,RRRE
   54   FORMAT%1HK,5HRR/RE,1X,1H#,E14.7¤
        PRINT55,RM
   55   FORMAT%1HK,8HESTIMATE,1X,4HMACH,1X,6HNUMBER,1X,1H#,E14.7¤
        PRINT56,PHT
   56   FORMAT%1HK,3HPHT,1X,1H#,E14.7¤
        READ66,R,TE,PAPC,G
   66   FORMAT%4F10.0¤
        PRINT67,TE
   67   FORMAT%1HK,4HEXIT,1X,11HTEMPERATURE,1X,1H#,E14.7¤
        PRINT68,PAPC
   68   FORMAT%1HK,5HPA/PC,1X,1H#,E14.7¤
        IF%GAS¤4,4,2
    2   READ3,GAMA
    3   FORMAT%F5.0¤
        PRINT 57
   57   FORMAT%1HK,5HUSING,1X,5HIDEAL,1X,3HGAS¤
        GO TO 8
    4   READ5,NT
    5   FORMAT%I4¤
        DO7I#1,NT
        READ6,HM%I¤,GAM%I¤
    6   FORMAT%2F10.7¤
    7   CONTINUE
        PRINT51
   51   FORMAT%1HK,5HUSING,1X,4HREAL,1X,3HGAS¤
    8   CONTINUE
   34   IF%GAS¤30,9,9
   30   DO31J#1,NT
        I#J
        IF%RM-HM%J¤¤33,32,31
   31   CONTINUE
   32   GAMA#GAM%I¤
        GO TO 9
   33   GAMA#GAM%I-1¤&%RM-HM%I-1¤¤*%GAM%I¤-GAM%I-1¤¤/%HM%I¤-HM%I-1¤¤
    9   FME#%2.0&%GAMA-1.0¤*RM*RM¤/%GAMA&1.0¤
        COM#%GAMA&1.0¤/%2.*%GAMA-1.0¤¤
        FME#RM*XP-FME**COM
        FPM#%2.0&%GAMA-1.0¤*RM*RM¤/%GAMA&1.0¤
        COM#%3.0-GAMA¤/%2.0*%GAMA-1.0¤¤
```

```
      FPM#XP-RM*%FPM**COM□
      DM#-FME/FPM
      RM#RM&DM
      DM#ABSF%DM□
      IF%DM-0.00001□10,10,34
   10 CONTINUE
      A#%GAMA-1.0□*%RM*RM-1.0□/%GAMA&1.0□
      A#SQRTF%A□
      A#ATANF%A□
      B#SQRTF%%GAMA&1.0□/%GAMA-1.0□□
      C#SQRTF%RM*RM-1.0□
      C#ATANF%C□
      VE#B*A-C
      RMEI#PEIPC**%%1.0-GAMA□/GAMA□
      RMEI#%2.0/%GAMA-1.□□*%RMEI-1.0□
      RMEI#SQRTF%RMEI□
      IF%GAS□36,35,35
   36 DO37J#1,NT
      I#J
      IF%RMEI-HM%J□□39,38,37
   37 CONTINUE
   38 GAMA#GAM%I□
      GO TO 35
   39 GAMA#GAM%I-1□&%RM-HM%I-1□□*%GAM%I□-GAM%I-1□□/%HM%I□-HM%I-1□□
   35 A#%GAMA-1.0□*%RMEI*RMEI-1.0□/%GAMA&1.0□
      A#SQRTF%A□
      A#ATANF%A□
      B#SQRTF%%GAMA&1.0□/%GAMA-1.0□□
      C#SQRTF%RMEI*RMEI-1.0□
      C#ATANF%C□
      VEI#B*A-C
      Y#1.0/RMEI
      UEI#ATANF%Y/SQRTF%1.0-Y*Y□□
      THEI#VEI-VE
      PHEI#THEI&UEI
C     CALCULATE THE ORIGIN OF THE LAST INTERNAL EXPANSION WAVE
      A1#%2.0/%GAMA&1.0□□*%1.0&0.5*%GAMA-1.0□*RMEI*RMEI□
      A1#A1**%%GAMA&1.0□/%2.0*%GAMA-1.0□□□
      B1#SINF%PHEI□
      RPRE#SQRTF%1.0-A1*B1/XP□
      XPRE#%RPRE-1.0□*COSF%PHEI□/SINF%PHEI□
      XN1#N1
      DRM#%RMEI-1.0□/XN1
      K#0
      XM#1.0
      PRINT17
  170FORMAT%1HK,4HMACH,10X,6HRX1/RE,8X,6HXX1/RE,8X,6HRX2/RE,8X,6HXX2/RE
     1,8X,5HPX/PC□
   44 IF%GAS□40,12,12
   40 DO43J#1,NT
      I#J
      IF%XM-HM%J□□41,42,43
```

```
   43 CONTINUE
   42 GAMA#GAM%I□
      GO TO 12
   41 GAMA#GAM%I-1□&%XM-HM%I-1□□*%GAM%I□-GAM%I-1□□/%HM%I□-HM%I-1□□
   12 A#%GAMA-1.0□*%XM*XM-1.0□/%GAMA&1.0□
      A#SQRTF%A□
      A#ATANF%A□
      B#SQRTF%%GAMA&1.0□/%GAMA-1.0□□
      C#SQRTF%XM*XM-1.0□
      C#ATANF%C□
      VX#A*B-C
      BX#PHT-1.570796-VX&ABSF%THEI□
      XLRE#2.0*RRRE*SINF%0.5*BX□
      PSI#3.1416-PHT&VX-0.5*%3.1416-BX□
      RX1RE#RPRE&XLRE*SINF%PSI□
      XX1RE#XPRE-XLRE*COSF%PSI□
      IF%K□62,60,62
   60 RX2RE#SQRTF%RX1RE*RX1RE&SINF%PHT□/XP□
      XX2RE#XX1RE&%RX2RE-RX1RE□*COSF%PHT□/SINF%PHT□
      GO TO 61
   62 UX#ATANF%1.0/%XM*SQRTF%1.0-%1.0/XM□**2□□□
      PHX#2.0*VEI-VE-VX&UX
      A2#%2.0/%GAMA&1.0□□*%1.0&0.5*%GAMA-1.0□*XM*XM□
      B2#0.5*%GAMA&1.0□/%GAMA-1.0□
      RX2RE#SQRTF%RX1RE*RX1RE&%A2**B2□*SINF%PHX□/XP□
      XX2RE#XX1RE&%RX2RE-RX1RE□*COSF%PHX□/SINF%PHX□
   61 PXPC#%1.0&0.5*%GAMA-1.0□*XM*XM□**%-GAMA/%GAMA-1.0□□
      PRINT13,XM,RX1RE,XX1RE,RX2RE,XX2RE,PXPC
   13 FORMAT%1HK,6E14.7□
      K#K&1
      IF%K-N1□14,14,15
   14 XM#XM&DRM
      GO TO 44
   15 PRINT16
   160FORMAT%1HK,8HINTERNAL,1X,7HPORTION,1X,2HOF,1X,3HTHE,1X,6HNOZZLE,1X
     1,2HIS,1X,8HCOMPLETE□
C     DESIGN OF EXTERNAL CONTOUR
      PRINT18
   180FORMAT%1HK,4HMACH,10X,5HRX/RE,9X,5HXX/RE,9X,5HPX/PC,9X,5HCFVAC,
     19X,3HSP.,1X,7HIMPULSE,2X,4HVAC.,1X,7HIMPULSE□
      UX#ATANF%1.0/%XM*SQRTF%1.0-%1.0/XM□**2□□□
      A#SQRTF%%GAMA-1.0□*%XM*XM-1.0□/%GAMA&1.0□□
      A#ATANF%A□
      B#SQRTF%%GAMA&1.0□/%GAMA-1.0□□
      C#SQRTF%XM*XM-1.0□
      C#ATANF%C□
      VX#B*A-C
      RXRE#%2.0/%GAMA&1.0□□*%1.0&0.5*%GAMA-1.0□*XM*XM□
      RXRE#RXRE**%%GAMA&1.0□/%2.0*%GAMA-1.0□□□
      RXRE#1.0-RXRE*SINF%VE-VX&UX□/XP
      RXRE#SQRTF%RXRE□
      XXRE#%1.0-RXRE□*COSF%VE-VX&UX□/SINF%VE-VX&UX□
```

80-80 CARD TO PRINTER

```
      C1#%2.0/%GAMA&1.0□□**%GAMA/%GAMA-1.0□□
      C2#SQRTF%%0.5*%GAMA&1.0□*XM*XM□/%1.0&0.5*%GAMA-1.0□*XM*XM□□
      SUMCG#GAMA*C1*C2*COSF%THEI□&XP*PXPC*%1.0-RXRE*RXRE□
      VT#TE*%1.0&0.5*%GAMA-1.0□*RM*RM□
      VT#GAMA*G*R*VT/%0.5*%GAMA&1.0□□
      VT#SQRTF%VT□
      VQ#TE*%1.0&0.5*%GAMA-1.0□*RM*RM□
      VC#1.0&0.5*%GAMA-1.0□*XM*XM
      VQ#GAMA*R*G*VQ/VC
      VQ#SQRTF%VQ□
      A#1.0&0.5*%GAMA-1.0□*XM*XM
      B#-GAMA/%GAMA-1.0□
      A#A**B
      C#1.0-PAPC*A*SINF%PHEI□/%XM*XM□
      D#COSF%THEI□&C/GAMA
      SUMIM#VQ*D/G
      CO#COSF%THEI□&1.0/GAMA
      SUMVA#VQ*CO/G
      PRINT19,XM,RXRE,XXRE,PXPC,SUMCG,SUMIM,SUMVA
   19 FORMAT%1HK,7E14.7□
      K1#1
      XN2#N2
      DRM#%RM-XM□/XN2
      XM#XM&DRM
      PRO#PXPC
      RXO#RXRE
   50 UX#ATANF%1.0/%XM*SQRTF%1.0-%1.0/XM□**2□□□
      IF%CAS□46,45,45
   46 DO49J#1,NT
      I#J
      IF%XM-HM%J□□47,48,49
   49 CONTINUE
   48 GAMA#GAM%I□
      GO TO 45
   47 GAMA#GAM%I-1□&%XM-HM%I-1□□*%GAM%I□-GAM%I-1□□/%HM%I□-HM%I-1□□
   45 A#SQRTF%%GAMA-1.0□*%XM*XM-1.0□/%GAMA&1.0□□
      A#ATANF%A□
      B#SQRTF%%GAMA&1.0□/%GAMA-1.0□□
      C#SQRTF%XM*XM-1.0□
      C#ATANF%C□
      VX#B*A-C
      RXRE# %2.0/%GAMA&1.0□□*%1.0&0.5*%GAMA-1.0□*XM*XM□
      RXRE#RXRE**%%GAMA&1.0□*0.5/%GAMA-1.0□□
      RXRE#SQRTF%1.0-RXRE*SINF%VE-VX&UX□/XP□
   65 XXRE#%1.0-RXRE□*COSF%VE-VX&UX□/SINF%VE-VX&UX□
      PXPC#%1.0&0.5*%GAMA-1.0□*XM*XM□**%-GAMA/%GAMA-1.0□□
      SUMCG#SUMCG&0.5*XP*%PRO&PXPC□*%RXO*RXO-RXRE*RXRE□
      A#GAMA/%GAMA-1.0□
      A#%0.5*%GAMA&1.0□□**A
      A#A*VT/%GAMA*G□
      B#0.5*A*XP
      SUMIM#SUMIM&B*%PRO&PXPC-2.0*PAPC□*%RXO*RXO-RXRE*RXRE□
```

```
      SUMVA#SUMVA&B*%PRO&PXPC¤*%RXO*RXO-RXRE*RXRE¤
      PRINT21,XM,RXRE,XXRE,PXPC,SUMCG,SUMIM,SUMVA
   21 FORMAT%1HK,7E14.7¤
      IF%K1-N2¤22,23,23
   22 XM#XM&DRM
      PRO#PXPC
      RXO#RXRE
      K1#K1&1
      GO TO 50
   23 PRINT 24
   240FORMAT%1HK,8HEXTERNAL,1X,7HPORTION,1X,2HOF,1X,3HTHE,1X,6HNOZZLE,1X
     1,2HIS,1X,8HCOMPLETE¤
      GO TO 101
      END
```