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THE DEVELOPMENT OF A HOMOGENEOUS NUMERICAL
OCEAN MODEL FOR THE ARCTIC OCEAN

by

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ABSTRACT:

A numerical ocean model driven by surface stress and a source-sink distribution is developed for a homogeneous ocean. Non-linearities, lateral friction and bottom friction are included. The basin shape can be varied to accommodate a large variety of configurations. Variable bathymetry and sources/sinks around the perimeter are included. The numerical scheme is conditionally stable and has second order accuracy in space and time.

A number of test cases are run to explore the dynamic significances of the various processes represented. The possible influence of these processes on the circulation of the Arctic ocean are discussed.
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Section I - Introduction and Report Summary

The general equations that describe the flow of both the atmosphere and the ocean are well known and are available to geophysicists for the investigation of a large variety of circulation problems. For many cases of interest analytic solutions to these general equations are not possible because of significant non-linearities in the equations, or because of irregular geophysical boundary configurations, or both. Although analytic solutions are not generally available it is often possible to obtain useful approximate solutions using numerical techniques. This numerical modeling has become a powerful tool for the investigation of both the ocean and the atmosphere.

Large scale numerical models available for the description of many features of the atmosphere and large portions of the oceans can be divided into roughly two groups. The first group is exploratory in nature and is used to investigate the significance of various physical processes. The prime emphasis is to understand just why a particular system responds as it does and what effects variations in forcing have on the outcome. The results from this type modeling may, or may not, be physically realistic but in all cases should elucidate the characteristics of the flow to be expected, its sensitivity to various inputs and significant correlations that are likely to exist. The second group of numerical models is generally more advanced and is designed for forecasting geophysical fluid flow on a more or less real time basis. The relative stage of refinement and the
large amounts of high quality data required for initial conditions has inhibited the development of these type models for the ocean and essentially all of the forecasting models routinely used deal with the atmosphere.

The object of this research was to begin a numerical exploration on the large scale circulation of the Arctic Ocean.

In the past numerical models of the Arctic Ocean have concentrated their attention on the sea ice that overlies most of the Ocean (Campbell, 1965) and tried to answer questions concerning the drift and climatological permanence of the pack ice (Maykut and Untersteiner, 1971). With the exception of Campbell's use of a simplified ocean model under an ice-layer model no significant effort has been directed towards a numerical study of the actual flow of the Arctic Ocean's waters.

At the onset of this research it must be admitted that very little is actually known about Arctic Ocean dynamics and that difficulties can be anticipated in deciding on appropriate boundary conditions and input parameters for the model. In many cases field data from the Arctic is lacking, or inadequate to give the required stress fields or inflow-outflow conditions with sufficient accuracy. This then demands that best estimates of boundary conditions serve as a tentative guide and that wide ranges of parametric inputs actually be investigated. The resulting numerical exploration yields considerable insight into the relative significance of various oceanographic parameters.

In addition to the uncertainties related to the lack of actual input data the development of a new numerical model has potential difficulties
inherent in the finite difference mathematic's that must be used. Both these problems can best be addressed by the careful development of the model from simple to more complex cases in a stepwise progression and with each step being checked against any available data (from the field or analytic considerations). This model is carried through this sort of genesis.

The first step in the model to be used for the Arctic assumes homogenous water and variable depth which is in some way similar to a model used by Holland (1976). The grid system used is based on a triangular plan similar to some systems used by Williamson (1968) and Sadourny, Arakawa and Mintz (1967). Both lateral and bottom friction are included in the model. The flow is driven by stress applied at the surface (simulating the wind or ice stress) and by source-sink distributions around the edge (that simulates major channels between the Arctic and other oceans). The details of the development for this first step in the model are given in the next section. The initial check out and experimentation with this first stage in the development of an Arctic Ocean circulation model has led to some interesting results. In particular: 1) negative curl introduced into the flow results in current patterns resembling the Beaufort Gyre; and 2) the Lomonosov Ridge (Fig. 1) acts like a dynamic block which may greatly increase the significance of the circulation caused by the source-sink distribution. There is some indication from field data reported by Muench (1970) and Thorndike (1971) that the processes indicated in the model have some counterpart in actual circulation observed
Figure 1. Chart of the Arctic Basin with the 100, 1000 and 2000 fathom depth contours drawn. Cross indicates location of the North Pole.
in the Arctic Ocean. A more detailed discussion of these results are presented in section three of this report.

Future extension and development of the model will be continued in stages. In the immediate future the present model will run with greatly increased resolution to delineate more of the complicated bathymetry of the Arctic Basin. After those results have been carefully explored the stratification appropriate to the Arctic Ocean must be introduced into the modeling. The results for the stratified ocean model will indicate how the next step (the inclusion of more complicate thermodynamic exchanges) can best be approached. A more detailed discussion of future modeling plans is given in section four of this report.

Section II - Development of the First Stage Numerical Model

When one considers the actual complexity of the circulation in the Arctic Ocean and the wide variety of numerical modeling techniques available for its study it is by no means obvious which path will lead to maximum returns. Each of the presently used numerical models has its advantages and limitations. In an attempt to address this question in a rational way it was decided to start with the simplest numerical model that could simulate what are thought to be the dominant forcing and geomorphology in the Arctic.

There can be little doubt that much of the large scale circulation of the Arctic Ocean is wind driven either directly, or indirectly through an ice cover that acts as some sort of coupling element (Campbell, 1965).
The details of how the ice cover couples the atmosphere to the ocean and to what extent it filters the time and space variations are unknown. There is however, a substantial effort directed towards this problem (Untersteiner, and Fletcher, 1971) and some progress can be anticipated. For the present model these interesting questions can not be treated in detail and thus the stress on the top of the ocean will be considered a known field, externally specified.

Studies by Coachman and Barnes (1961, 1963), Aagaard (1966) all indicate that there may be dynamically significant exchange between the Arctic Basins and the adjacent portions of the world ocean. For this reason the numerical model incorporates sources and sinks of water around its perimeter to simulate the major channels between the Arctic and the adjacent parts of the ocean.

Looking at Figure 1., it is seen that a large fraction of the Arctic is covered by the relative shallow Siberian Shelf, while the deeper portion of the Arctic Ocean is divided into two basins (both over 4000 meters deep) by the Lomonosov Ridge. In an attempt to retain some of the dynamical effects caused by these large bathymetric variations the model includes variable depth.

It is likely that the density variations in the Arctic Ocean effect the flow. For example a full treatment of the movement in the Atlantic layer (Coachman and Barnes 1963) will certainly require a baroclinic model. On the other hand there are some actual current measurements from the Arctic (Nikitin and Demyanov, 1965) (Galt, 1967) (Coachman, 1969) that indicate
a substantial barotropic, or depth independent component to the currents. This coupled with a consideration of the great increase in complexity required for variable density models suggests that for the initial numerical exploration a homogeneous or barotropic formulation be used.

The circulation in the Arctic is not well enough known to come up with an accurate appraisal of the significance of frictional forces. It seems likely that near source-sink points lateral friction could effect the flow. Over the large area covered by the Siberian Shelf it is quite possible that bottom friction might also be significant. Accordingly both lateral and bottom friction were included in the model and it was anticipated that some range of frictional parameters would be investigated to test for significance.

To develop a model with the characteristics described above the following integrated form of the equations of motion are used:

\[
\frac{D u}{D t} - f v = - \frac{1}{\rho} \frac{\partial p}{\partial x} + K v^2 u - \frac{R u}{h} + \frac{\tau_x}{\rho h} \\
\frac{D v}{D t} + f u = - \frac{1}{\rho} \frac{\partial p}{\partial y} + K v^2 v - \frac{R v}{h} + \frac{\tau_y}{\rho h}
\]  

(1) (2)

Where the dependent variables \( u \) and \( v \) are the horizontal components of velocity which are independent of depth. The density, \( \rho \), is a constant. \( \tau_x \) and \( \tau_y \), the components of the wind stress, \( h \), the depth, and \( f \), the coriolis parameter, are functions of position. \( K \) and \( R \) are constants that specify the effectiveness of the horizontal and vertical frictional forces respectively.
In addition to these equations of motion we have the continuity equation:

$$\frac{\partial}{\partial x} (hu) + \frac{\partial}{\partial y} (hv) = 0 \quad (3)$$

A transport stream function is introduced such that:

$$-hu = \frac{\partial \psi}{\partial y} \quad (4)$$

$$hv = \frac{\partial \psi}{\partial x}$$

The pressure can be eliminated from equations (1) and (2) and after some manipulation the following vorticity equation is obtained:

$$\frac{\partial \xi}{\partial \tau} - (\nabla \times \psi \vec{E}) \cdot \nabla \left( \frac{\xi + f}{h} \right)$$

$$= K \nabla^2 \xi - \frac{R}{h} \left[ \nabla + \nabla \times \psi \cdot \nabla \left( \frac{1}{h} \right) \right] + \nabla \times \left( \frac{\tau}{\rho h} \right) \quad (5)$$

where:

$$\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

$$\nabla = \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y}$$

$$\vec{\tau} = \tau_x \vec{i} + \tau_y \vec{j}$$

and \( \vec{i}, \vec{j}, \vec{k} \) are the unit vectors in the right handed \( x, y, z \) coordinate system.

From the above the following relationship between \( \xi \) and \( \psi \) is obtained:

$$\nabla \left( \frac{1}{h} \nabla \psi \right) = \xi \quad (6)$$

Equations (5) and (6) can now be solved for the vorticity and stream
function provided that the proper initial and boundary conditions are
given. In particular the following must be specified:

a) $\psi$ - given within the region of interest at $t = 0$

b) $\psi$ - given on the boundary of the region for all time

c) $\xi$ - given within the region of interest at $t = 0$

Note that condition b) is equivalent to specifying the source-sink distri-
bution around the edge of the model.

Equations (5) and (6) can be non-dimensionalized by introducing the
following new variables:

$$
t = t'\left(\frac{1}{F}\right) \quad f = f'F
\quad x = x' \Delta \quad y = y' \Delta
\quad h = h'D
\quad \psi = \psi'\psi_o
\quad \tau = \tau'\left(\frac{\rho F \psi_o}{\Delta}\right)
$$

(7)

Where the primed quantities are all non-dimensional, $F$ is the average value
of the Coriolis parameter, $\Delta$ is the finite difference grid spacing, $D$ is
the average depth of the model and $\psi_o$ is equal to the max range of stream
function on the boundary of the model, or the maximum value expected for
the wind driven portion of the flow, which ever is the most convenient.

Using the new variables defined above equations (5) and (6) become;

$$
\begin{aligned}
\frac{\partial \xi'}{\partial t} = (\nabla \times \psi') \cdot \nabla \left( \frac{\nabla \xi' + f'}{h'} \right) \\
= \frac{g}{h'} \nabla \cdot \left[ (\xi' + \nabla \psi') \cdot \nabla \left( \frac{1}{h'} \right) \right] + \nabla \times \left( \frac{\tau'}{h'} \right)
\end{aligned}
$$

(8)

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\[ \nabla \left( \frac{1}{h'} \nabla \psi' \right) = \xi' \]  \hspace{1cm} (9)

Where:

\[ \alpha = \frac{\psi_0}{\Delta^2 D_F} \]

\[ \beta = \frac{K}{\Delta^2 F} \]

\[ \gamma = \frac{R}{DF} \]

These three non-dimensional parameters govern the character of the solution. For example setting \( \alpha = 0 \) removes the non-linear advective terms from the model. The size of \( R \) determines how important lateral friction is in the solution and \( \gamma \) scales the importance of bottom friction.

The finite difference grid that equations (8) and (9) are solved on is made up of a one dimensional array of \( N \times M \) points. They are arranged as shown in figure 2. Grid points are numbered sequentially, left to right starting in the top row. This means that grid point in the neighborhood of the point \( L \) are given as shown in figure 2.

Two additional one dimensional arrays are used to specify the extent of the interior domain. These are integer arrays labeled IA and JA of dimension \((M-2)\) and are defined so that a sweep of interior model points is obtained via the following algorithm.

\[ \text{DO 2 } K=1, M-2 \]

\[ I = IA(K) \]

\[ J = JA(K) \]

\[ \text{DO 1 } L = I, J \]

1 Statement on Interior Point (L)

2 Continue
Figure 2. Grid pattern used in finite difference scheme and numbering system used for calculational molecule.
The depth in the model is specified in terms of an average depth and an array of deviations from this average. The amount of bathymetry to be used in any particular run is specified by a depth factor which is input at run time. The following depth variables are thus defined.

\[ \text{HM} = \text{average water depth} \]

\[ \text{HFAC} = \text{depth factor} \]

\[ H \text{ (DIMENSIONED TO N x M)} = \text{array of depth deviations}. \]

The actual depth used in any particular run is calculated with the following algorithm

\[ \text{DO 3 I = 1, NM} \]

\[ 3 \ H(I) = \text{HM + HFAC} \times H(I) \]

Obviously \( \text{HFAC} = 0 \) gives a constant depth model and \( \text{HFAC} = 1.0 \) includes all of the bathymetry. Any intermediate fraction is also possible if experimentation suggests such a case would be of interest.

To integrate equation (8) the three level Adams - Bashforth method is used. That is, writing equations (8) as:

\[ \frac{\partial \xi'}{\partial t} = g' \]

we use:

\[ \xi'(t' + \Delta t') = \xi'(t') + \left[ \frac{3}{2} g'(t') - \frac{1}{2} g'(t' - \Delta t') \right] \Delta t' \]

(10)

To use this equation we must write \( g' \) in finite difference form. This will be done term by term starting with \( g' \) written as:

\[ g' = (\nabla \times \psi') \cdot \nabla \left( \frac{\alpha \xi' + \xi'}{h'} \right) + \beta \nabla^2 \xi' 
- \nabla \left[ \xi' + \psi' \cdot \nabla \left( \frac{1}{h'} \right) \right] + \nabla \times \left( \frac{\vec{r}}{h'} \right) \]

(11)
The first term on the right hand side of equation (11) represents the net rate of potential vorticity advected into a unit area. To approximate this for the hexagon centered on the point L we write: (all quantities are non dimensional and primes have been omitted for simplicity)

\[
(\nabla \times \psi_i^* \cdot \nabla \left( \frac{Q_n + f}{h} \right) = \frac{2}{3\sqrt{3}} \left[ (\psi_{L-N+1} - \psi_{L+1})^2 \frac{PV_{L-N+1} + PV_{L+1}}{2} \right] \\
+ (\psi_{L-N} - \psi_{L-N+1}) \frac{PV_{L-N} + PV_{L-N+1}}{2} + (\psi_{L-1} - \psi_{L-N}) \frac{PV_{L-1} + PV_{L-N}}{2} \\
+ (\psi_{L+N-1} - \psi_{L-1}) \frac{PV_{L+N-1} + PV_{L-1}}{2} + (\psi_{L+N} - \psi_{L+N-1}) \frac{PV_{L+N} + PV_{L+N-1}}{2} \\
+ (\psi_{L+1} - \psi_{L+N}) \frac{PV_{L+1} + PV_{L+N}}{2} 
\]

(12)

Where:

\[ PV_i = (C1)(\xi_i) + f_i \]

and

\[ C1 = \alpha = \frac{\psi_0}{\Delta^2 DF} \]

The second term on the right hand side of equation (11) represents the diffusion of vorticity horizontally and in finite difference form becomes:

\[
\beta \nabla^2 \xi = (C2) \left[ \xi_{L+1} + \xi_{L-N+1} + \xi_{L-N} + \xi_{L-1} \right. \\
+ \xi_{L+N-1} + \xi_{L+N} - (6)\xi_L \left. \right] 
\]

(13)

where

\[ C2 = \frac{2\beta}{3} = \frac{2K}{3\Delta^2 F} \]

the third term on the right hand side of equation (11) represents the
dissipation of vorticity through bottom friction and is written as:

\[
\mathbf{v} \cdot \nabla \psi \cdot \nabla (\frac{1}{h}) = \frac{C^3}{h_L} \{ \xi_L + \frac{1}{4}(\psi_{L+1} - \psi_{L-1}) \\
+ \frac{1}{2}(\psi_{L-N+1} - \psi_{L-N} - \psi_{L-N} + \psi_{L+N}) \}
\]

\[
= \left[ \frac{1}{h_{L+1}} - \frac{1}{h_{L-1}} + \frac{1}{2} \left( \frac{1}{h_{L-N+1}} - \frac{1}{h_{L-N}} - \frac{1}{h_{L-N}} + \frac{1}{h_{L+N}} \right) \right] \\
+ \frac{3}{8} \left[ \psi_{L-N+1} - \psi_{L-N} + \psi_{L-N} - \psi_{L+N} \right] \\
\left[ \frac{1}{h_{L-N+1}} - \frac{1}{h_{L+N-1}} + \frac{1}{h_{L-N}} - \frac{1}{h_{L+1}} \right]
\]

(14)

where:

\[ C^3 = \gamma = \frac{R}{DF} \]

The last term on the right hand side of equation (11) is the torque added per unit time by wind stress. This is assumed constant in time and calculated only once at the beginning of the program using the following finite difference form.

\[
\nabla \times \left( \frac{\mathbf{r}}{h} \right) = \frac{1}{2} \left\{ \left( \frac{G^2}{h} \right)_{L+1} - \left( \frac{G^2}{h} \right)_{L-1} \right\} \\
- \left( \frac{1}{\sqrt{3}} \right) \left\{ \left( \frac{G^1}{h} \right)_{L-N+1} - \left( \frac{G^1}{h} \right)_{L-N} + \left( \frac{G^1}{h} \right)_{L-N} - \left( \frac{G^1}{h} \right)_{L+N-1} \right\}
\]

(15)

Where \( G^1 \) is the component of the wind stress in the direction from \( L \) to \( L + 1 \) and \( G^2 \) is the component of the wind stress in a direction 90 degrees to the left of \( G^1 \).

Equations (12) through (15) are used in equation (11) which in turn is used in the time integration scheme defined in equation (10). Once the vorticity has been obtained for a new time step equation (9) is solved by a
successive overrelaxation technique.

The algorithm used is a slight modification of the scheme suggested by Winslow (1961). The scheme used is as follows:

A residual is calculated:

\[
\text{RES} = \nabla \left( \frac{1}{h} \nabla \psi \right) - \zeta
\]

\[
= \left( \frac{4}{3} \right) \left[ \frac{\psi_{L+1}}{(h_{L+1} + h_L)} \right] + \frac{\psi_{L-N+1}}{(h_{L-N+1} + h_L)} + \frac{\psi_{L-N}}{(h_{L-N} + h_L)}
\]

\[
+ \frac{\psi_{L-1}}{(h_{L-1} + h_L)} + \frac{\psi_{L+N-1}}{(h_{L+N-1} + h_L)} + \frac{\psi_{L+N}}{(h_{L+N} + h_L)} - \psi_{L}(HFAC_L) - \xi_L
\]

(16)

where:

\[
HFAC_L = \left[ \frac{1}{h_{L+1} + h_L} \right] + \left[ \frac{1}{h_{L-N+1} + h_L} \right] + \left[ \frac{1}{h_{L-N} + h_L} \right]
\]

\[
\left[ \frac{1}{h_{L-1} + h_L} \right] + \left[ \frac{1}{h_{L+N-1} + h_L} \right] + \left[ \frac{1}{h_{L+N} + h_L} \right]
\]

The residual is then normalized using the coefficient of \( \psi_L \) in equation (16), i.e.,

\[
dpsi = \left( \frac{4}{3} \right) \left( \frac{1}{HFAC_L} \right) \text{RES}
\]

and the relaxation is then given by:

\[
\psi_L = \psi_L + (R)dpsi_L
\]

(17)

A number of tests were run to estimate the optimal value for \( R \) and over a wide range of cases a value of 1.48 was indicated. This value was then used throughout the modeling efforts reported here.

The typical experiment anticipated for the model will be to apply some
stress field to a static ocean. The model ocean will then spin-up, or
develop a circulation pattern that will generally approach a steady state
providing an appropriate balance of torque is possible and that the
applied stress field is steady. The time dependent development of the
steady state circulation and to some extent the final flow pattern will
depend on the characteristics of the planetary waves that make up the
transients in the model. For this reason it is of some interest to look
at the propagation characteristics of these waves within the model and
to estimate the errors introduced by the finite difference scheme.

To estimate how the model transients will respond a simple, free,
linear Rossby wave will be considered. For this case the vorticity
equation (5) reduces to:

$$\frac{\partial}{\partial t} \left( \frac{1}{h} \nabla^2 \psi \right) = -hv \cdot \nabla \left( \frac{f}{h} \right)$$  \hspace{1cm} (18)

A wave form of the solution is assumed and near by values used in the
computational molecule (Fig. 2) taken as follows:

$$\psi_L = \psi_0 \epsilon^{i(kx + ly - \omega t)}$$

$$\psi_{L+1} = \epsilon \frac{i \Delta}{\sigma} \psi_L$$

$$\psi_{L-N+1} = \epsilon \frac{i \Delta}{\sigma} (k + \sqrt{3} \ell) \psi_L$$

$$\psi_{L-N} = \epsilon \frac{i \Delta}{\sigma} (-k + \sqrt{3} \ell) \psi_L$$

$$\psi_{L-1} = \epsilon^{-i k \Delta} \psi_L$$  \hspace{1cm} (19)
\[ \psi_{L+N-1} = \epsilon \frac{i\Delta}{2} (-k - \sqrt{3}j) \psi_{L} \]
\[ \psi_{L+N} = \epsilon \frac{i\Delta}{2} (k - \sqrt{3}j) \psi_{L} \]

Where \( x \) and \( y \) refer to position in a right handed co-ordinate system superimposed on the finite difference grid with the positive \( x \)-axis in the direction form \( L \) to \( L + 1 \). It can also be assumed with no loss of generality that the \( x \) and \( y \) axes are directed east and north respectively. This then gives:

\[ \frac{\partial f}{\partial x} = 0 ; \quad \frac{\partial f}{\partial y} = \beta^* \quad (20) \]

To obtain a reasonable estimate of the phase velocity for the Rossby waves in the model the time differencing can be assumed exact and the depth assumed constant. Using equations (19) and (20) with the finite difference forms given in equations (12) and (13), the vorticity equation given in (18) becomes, after some simplification.

\[ \frac{-i\omega}{3h\Delta^2} \left[ \cos k\Delta + 2 \cos \left( \frac{\sqrt{3}k\Delta}{2} \right) \cos \left( \frac{k\Delta}{2} \right) - 3 \right] \]
\[ = \frac{-2i\beta^*}{\Delta} \left[ \sin k\Delta + \sin \left( \frac{k\Delta}{2} \right) \cos \left( \frac{\sqrt{3}k\Delta}{2} \right) \right] \]

Which gives

\[ \omega = \frac{\beta^*\Delta}{2} \left[ \frac{\sin (k\Delta) + \sin \left( \frac{k\Delta}{2} \right) \cos \left( \frac{\sqrt{3}k\Delta}{2} \right)}{\cos (k\Delta) + 2 \cos \left( \frac{k\Delta}{2} \right) \cos \left( \frac{\sqrt{3}k\Delta}{2} \right) - 3} \right] \]

For small \( \Delta \) the trig functions can be expanded using small angle approximations.
This gives:

\[
\omega = -\beta^* \left[ \frac{k - \frac{1}{8} (k^3 + k^2)(\Delta^2)}{k^2 + k^2 + \left(\frac{k^4}{18} + \frac{k^2 \ell^2}{8}\right)(\Delta^2)} \right]
\]

And from this dispersion relation it can easily be seen that the \(x\) component of the phase velocity is:

\[
C_x = \frac{\omega}{k} = -\beta^* \left[ \frac{1 - \frac{1}{8} (k^3 + k^2)(\Delta^2)}{k^2 + k^2 + \left(\frac{k^4}{18} + \frac{k^2 \ell^2}{8}\right)(\Delta^2)} \right]
\]

(21)

Clearly as \(\Delta\) goes to zero this goes to the exact form

\[
C_x = -\frac{q^*}{k^2 + \ell^2}
\]

and the error term is second order in \(\Delta\). This then suggests that the transient wave solutions that develop in the model will be accurately represented to second order.

The actual fortran program used for the model and an explanation of the input data required is given in an appendix.

Section III - Preliminary Results

The characteristics of the model described in the last section have been investigated in a series of tests designed to check out the numerical properties of the solutions. In a number of cases it was also possible to illustrate the physical significance that the processes represented might have on the circulation in the Arctic Ocean.

The first test was to check the part of the program that handled the
Figure 5. Streamlines of source-sink driven flow with uniform negative vorticity in the interior and uniform depth.
relaxation of the stream function. A basin shape was chosen that
roughly covered the deeper portion of the Arctic Ocean (Fig. 3). Repre-
sentative source-sink distributions were specified as follows:
2 × 10⁶ m³ sec⁻¹ flow in across the Chukchi Sea, 7 × 10⁶ m³ sec⁻¹
flow in spread out on either side of Franz Josef Land, 1 × 10⁶ m³ sec⁻¹
flow out through M'Clure Strait, 1 × 10⁶ m³ sec⁻¹ flow out into the
Lincoln Sea, and 7 × 10⁶ m³ sec⁻¹ flow out into the East Greenland
current. With a flat bottom the equations governing the flow in the in-
terior reduce to:
\[ \nabla^2 \psi = 0 \]
Solving this for the specified boundary conditions gives the flow indi-
cated in Figure 4.

The next test case introduced some finite vorticity into the above
equation. For the initial testing this was simply specified as a constant.
Thus the governing equation becomes:
\[ \nabla^2 \psi = h \xi_0 \]
This obviously corresponds to the artificial case where equation 8 has
gone to a steady-uniform solution for the vorticity. Although this is not
very realistic the results are interesting and shown in Figure 5 for the
case where the vorticity in the flow is constant and negative as one might
expect to get from the stress field developed by the polar easterlies.
Circulation resembling the Beaufort Gyre develops. Qualitatively the
results show a striking resemblance to the very viscous solution
for ice drift presented by Campbell (1965, Fig. 7c,d).

The next set of experiments with the model were to check out the time dependent vorticity equation (8) in conjunction with relaxation of the stream function (Eq. 9). To do this a basin roughly the shape of the Arctic was specified. Non-linear terms and bottom friction were not included (\(\alpha = \gamma = 0\)). A uniform negative stress curl was applied to the water that was initially at rest. The Coriolis parameter was assumed constant.

For the first series of runs made on this configuration the depth was assumed constant. Under these conditions the model would spin-up forming a symmetric clock-wise circulation pattern. The steady state solution represented a balance between the torque added by the wind and that which was lost through lateral friction. The magnitude of the steady state solution and the spin-up time of the model depended on the magnitude of the frictional coefficient (\(\beta\)) that was used. It is interesting to note that for this series the vorticity equation reduces to:

\[
\frac{\partial \xi}{\partial t} = \alpha \frac{\partial^2 \xi}{\partial z^2} + \frac{1}{h} (\mathbf{v} \times \tau)
\]  

(22)

With a steady wind stress this equation has no wave solutions and the appropriate von Neumann type stability condition (Richtmyer and Morton, 1967) will be of the form:

\[ \delta t \approx \frac{1}{4} \]  

(23)

(Assuming the Adams-Bashforth scheme is used for time differencing.)
Figure 6. Streamlines of flow driven by uniform stress curl in an irregular shaped ocean with a central ridge. (The ridge runs from top to bottom in the figure.)
Where $t$ is the non-dimensional time step in half pendulum days. This is relatively weak restriction in that with more or less realistic geophysical parameters time steps of a number of days are possible. This was confirmed with the model.

The second series run on this configuration was designed to test the models response to variable depth. In this series a ridge of smooth profile was placed across the basin. The appropriate form of the vorticity equation was

$$\frac{\partial \xi}{\partial t} - f(\nabla \times \psi) \cdot \nabla \left(\frac{1}{h}\right) = \beta \psi^2 \xi + \nabla \times \left(\frac{\mathbf{T}}{h}\right)$$  \hspace{1cm} (24)

Under these conditions topographic Rossby waves are possible (Veronis, 1966) and the von Neumann stability criterion takes on the more restrictive form: (Once again assuming an Adams-Bashforth scheme for the time differences)

$$\frac{Ct}{\Delta t} \leq \text{constant} \approx \frac{1}{2}$$  \hspace{1cm} (25)

Where $C$ is the phase speed of the topographic Rossby waves. This general behavior was again confirmed by the model.

In this series the spin-up of the model begins as before, but the variable depth acts in many ways analogously to a variable $F$ in a flat bottom ocean and western boundary type currents can develop. These stronger boundary currents develop not on the western side of the ocean basin as in Munk's model (Munk, 1950) but rather to the left of an observer looking from deep to shallow water. Figure 6 shows a typical solution...
for this series. The ridge is seen to divide the flow into two cells with regions of intensified currents in each half.

During the initial spin-up for this series transient wave patterns were seen to propagate along the ridge. It is quite likely that the real time dependent stress fields applied to the Arctic ocean will result in transients of this form being propagated along the Lomonosov Ridge (Fig. 1).

An additional series of tests were run to investigate the interaction of source-sink terms with the bathymetry and the formulation used to represent bottom friction. For this series the stress applied at the surface was zero. The source-sink distribution was simply to have flow in through the Bering Straits and out through the East Greenland area. The bathymetry was again the smooth ridge described in the previous series of tests.

The first test in this series assumed $\alpha = \beta = \gamma = 0$. Thus there was no friction, or non-linear terms. The appropriate form of the vorticity equation that was:

$$\frac{\partial \vec{\xi}}{\partial t} - (\nabla \times \vec{\psi}) \cdot \nabla \left( \frac{f}{h} \right) = 0 \quad (26)$$

This equation shows the interesting result that the only steady flow possible is when the stream lines are parallel to lines of constant $f/h$. Thus the steady-state flow can not cross the ridge and any source-sink distribution that demands cross ridge flow will not lead to a steady circulation. The numerical model shows these characteristics with
circulation continuing to increase with time. A strong negative (clockwise circulation builds up on the in-flow side of the ridge and a corresponding positive circulation develops on the outflow side of the ridge. This is similar to the effect one might get if the water simply piled up on the in-flow side and drained out on the out-flow side satisfying the overall continuity requirements for the ocean but with minimum cross-ridge flow.

This has an important implication for the circulation in the Arctic. If the source-sink component of the flow is to be barotropic and geostrophic then flow can not cross the Lomonosov ridge. This means that continuity must be satisfied separately for both basins in the Arctic. There will be a strong tendency for flow that enters on the Canadian side of the Arctic to exit on the same side. Field data from the Arctic suggest that this may in fact be the case (Coachman, 1970). Thus one might expect that flow through the Bering Straits and the Canadian Archipelago should be strongly correlated.

The next runs in this series included the non-linear terms and lateral friction which gave the following governing vorticity equation:

$$\frac{\partial \xi}{\partial t} - (\nabla \times \psi \nabla) \cdot \nabla \left( \frac{C + f}{h} \right) = \beta \nabla \xi$$

For this case both frictional and non-linear boundary layers are present with cross ridge flow taking place and a steady-state solution is possible. For all reasonable values of $\alpha$ and $\beta$ the interior of the flow is still essentially geostrophic and the cross ridge flow takes place in boundary layers
Figure 7. Streamlines of flow driven by a simple source-sink distribution for an irregular shaped ocean with a central ridge. (The ridge runs from the top to the bottom in the figure.)
where the ridge intersects the sides of the basin. (Fig. 7)

A final case run in this series included the bottom friction terms. It can be anticipated that bottom friction would be effective in causing cross bathymetry flow. In particular the

$$\frac{\gamma}{h} \nabla \psi \cdot \nabla \left( \frac{1}{h} \right)$$

term will have a tendency to turn flow towards shallow water. This is in fact what the model showed. Steady state flows resembled figure 7 for very small values of $\gamma$ and showed a smooth transition to stream line patterns resembling simple hydraulic flow (similar to Fig. 4) as the value of $\gamma$ was increased.

In the deeper interior of the Arctic Ocean the flow must be essentially geostrophic, but it seems likely that frictional effects must be significant over large portions of the Siberian shelf. In the vicinity of the Lomonosov Ridge both friction and non-linear effects may be significant.

Section IV - Future Model Plans

The next stages in the development and use of this model are well under way at the time of this writing.

The first extension is to use the model with greatly increased resolution and the actual bathymetric variation of the Arctic basins. With this configuration realistic source-sink distributions are investigated. A wind stress field obtained from Felezenbaum's pressure data (Felezenbaum, 1958) is used to drive the flow and a number of runs are
anticipated to investigate the effects of variations in $\alpha$, $\beta$ and $\gamma$.

The next series of tests will use the same basic model configuration but the stress field will be obtained by applying Felezenbaum's pressure data to Campbell and Rasmussen's (1971) ice model and then using the stress field from the bottom of the ice to drive the ocean model. Again a series of runs are anticipated to investigate variations in parameters.

Another series of runs is planned for the model that will have even higher spacial resolution and definitions of the bathymetry. In this series the ocean will be driven by a time dependent wind stress field that is made up of random frequency components. The dependent variables at a number of locations will be spectral analyzed with the intension of obtaining normal mode frequencies for the Arctic basins.

The next stage in the development of model exploration for the Arctic will be the formulation of a baroclinic model. The first set of experiments anticipated for this formulation will be the investigation of the dynamics associated with the circulation of the Atlantic Layer (Coachman and Barnes, 1963).
References


Felzenbaum, A. I. (1958). The theory of the steady drift of ice and the calculation of the long period mean drift in the central part of the Arctic basin. Problems of the North, 2, 5-15, 13-44.


Appendix - Use of Computer Program

The ocean model program has been written in FORTRAN - IV and a listing of the program is included in the end of this appendix. To use the program an appropriate data deck must be placed after the program. The composition of the data deck must be as follows:

1) N, M, NB FORMAT (313) - this is a single card where N and M are the dimensions of the grid system (ref. Fig. 2) and NB is the number of boundary points.

2) IA FORMAT (2013) - this is a grid number list of the first interior point of each line down the left hand side of the model.

3) JA FORMAT (2013) - this is a grid number list of the last interior point of each line down the right hand side of the model.

4) LV, BLANK FORMAT (21A1) - this is a single card with the code to be used in the graphical output of the program. (Ref. subroutine GRAOUT)

5) HM, FAC FORMAT (2F6.1) - this is a single card where HM is the reference depth and FAC is the fraction of the bathymetry that is desired in the model.

6) H FORMAT (12F6.4) - this is a set of cards that has the normalized depth variation for each grid point.

7) F FORMAT (12F6.4) - this is a set of cards that has the normalized values of the Coriolis parameter for each grid point.
8) I FORMAT (13) - this is a single card with the exponential part of the values to be used for the wind stress.

9) G1 FORMAT (12F5.3) - this is a set of cards with the significant figures part of the wind stress in the direction of the positive axis, i.e. from point L to L + 1 (Ref. Fig. 2).

10) G2 FORMAT (12F5.3) - this is a set of cards with the significant figures part of the wind stress in the direction 90° to the left of the positive axis.

11) IBC FORMAT (6(6I1, I3)) - this is a set of cards that contains the information required to calculate boundary values of vorticity. For each boundary point a six digit code and the grid number are given. The six digit code refers to the neighboring points in a counter clockwise order starting with L + 1. The following code is used:
0 - exterior point, 1 - interior point (free slip B.C.), 2 - interior point (no slip B.C), 3 - boundary point along streamline, 4 - boundary point across a streamline. Boundary points at sources are not included in the IBC list.

12) C1, C2, C3, DT, TOUT, TMAX, R, CON, 1PUNCH FORMAT(6E10.3, F4.2, E10.3, 12) - this is a single card that contains the run parameters. C1, C2 and C3 are defined in section two of this report. DT is the time step. TOUT is the time interval at which output of the dependent variables is desired. TMAX is the maximum time the model is to run. R is the SOR parameter to be used for the relaxation of the
stream function. CON is the absolute convergence limit to be used for the relaxation of the stream function. IPUNCH is a flag that should be non-blank if the final values for the dependent variables are required as a source deck for future runs.

13) S FORMAT (6E12.5) - this is a set of cards that contain the initial values for the stream function at each grid point.

14) V FORMAT(6E12.5) - this is a set of cards that contain the initial values for the vorticity at each grid point.

15) G1 FORMAT(6E12.6) - this is a set of cards that contain the initial values of the local time rate of chance of the vorticity at time -(-DT).
NUMERICAL OCEAN MODEL PROGRAM - DEVELOPED MAY 1971  J. A. GALT

ALL OF THE ARRAYS USED IN THE PROGRAM ARE DIMENSIONED HERE. IA(M2) AND JA(M2) SPECIFY THE SHAPE OF THE BASIN. H(NM), F(NM) AND T(NM) ARE USED FOR THE INDEPENDENT VARIABLES WATER DEPTH, CORIOLIS PARAMETER AND WIND STRESS CURL RESPECTIVELY. THE DEPENDENT VARIABLES ARE ASSOCIATED WITH S(NM) FOR THE STREAM FUNCTION AND V(NM) FOR THE VORTICITY. G1(NM) AND G2(NM) ARE CALCULATIONAL ARRAYS USED FOR TEMPORARY STORAGE AND L0((2*N+M-2)*M) ARE USED FOR THE OUTPUT OF ARRAY DATA. IBC(7,NB) IS AN ARRAY THAT CONTAINS THE COEFFICIENTS NEEDED TO CALCULATE BOUNDARY VALUES FOR THE VORTICITY. RES(NM) IS AN ARRAY USED IN THE VARIABLE DEPTH PART OF THE STREAM FUNCTION RELAXATION. LA(6) IS A STORAGE ARRAY USED DURING THE CALCULATION OF VORTICITY ON THE BOUNDARY POINTS. HC1(NM) AND HC2(NM) ARE COEFFICIENT ARRAYS USED IN THE CALCULATION OF BOTTOM STRESS.

DIMENSION IA(5),JA(5),H(49),F(49),T(49),G1(49),G2(49),LV(20),
E LO(133),IBC(7,18),RES(49),V(49),S(49),LA(6),
E HC1(49), HC2(49)

THE PROGRAM STARTS BY READING IN THE SIZE OF THE MAJOR ARRAY WHICH WILL BE N BY M POINTS AND THE NUMBER OF BOUNDARY POINTS NB

READ(5,100) N,M,NB

100 FORMAT(3I13)

SIZE PARAMETERS ARE NOW CALCULATED. NM IS THE TOTAL NUMBER OF GRID POINTS AVAILABLE TO THE DEPENDENT VARIABLES. NOUT IS THE NUMBER OF POINTS IN THE OUTPUT ARRAY. AND, M2 IS THE SIZE OF THE ARRAYS USED TO SPECIFY THE BASIN SHAPE.

NM = N*M
NOUT = (2*N+M-2)*M
M2 = M-2

THE ARRAYS THAT SPECIFY THE BASIN SHAPE ARE NOW READ IN

READ(5,101) IA
READ(5,101) JA

101 FORMAT(20I3)

THE ALPHA CODE USED IN THE OUTPUT ARRAYS IS READ IN

READ(5,102) LV,BLANK

102 FORMAT(21A1)
WRITE(6,103)
103 FORMAT(1H1,22H NUMERICAL OCEAN MODEL ///, 0049
E 19H THE BASIN SHAPE IS ///)
DO 2 K = 1,M2 0050
1 = IA(K) 0051
J = JA(K) 0052
DO 1 L = 1,J 0053
1 = I,L 0054
H(L) = FLOAT(L) 0055
2 CONTINUE 0056
CALL GRAOUT(LV,LO,NOUT,H,NM,IA,JA,M2,N,BLANK) 0057
C C THE REFERENCE DEPTH HM AND THE DEPTH FACTOR FAC ARE READ 0058
IN. AFTER THIS THE NORMALIZED DEPTH VARIATIONS ARE 0059
READ IN AND THE WORKING DEPTHS CALCULATED. 0060
104 FORMAT(2F6.1) 0061
READ(5,104) HM,FAC 0062
105 FORMAT(12F5.3) 0063
106 FORMAT(1H1,25H THE REFERENCE DEPTH IS = ,F8.2,7H METERS,// 0064
E 22H THE DEPTH FACTOR IS = ,F6.3///) 0065
WRITE(6,107) 0066
107 FORMAT(40H THE ARRAY OF NORMALIZED DEPTH VALUES IS ///) 0067
WRITE(6,108) H 0068
108 FORMAT(12F8.3) 0069
WRITE(6,109) 0070
109 FORMAT(18H BOTTOM TOPOGRAPHY ///) 0071
CALL GRAOUT(LV,LO,NOUT,H,NM,IA,JA,M2,N,BLANK) 0072
C C THE NORMALIZED VALUES OF THE CORIOLIS PARAMETER ARE NOW READ IN 0073
READ(5,110) F 0074
110 FORMAT(12F6.4) 0075
WRITE(6,111) 0076
111 FORMAT(1H1,46H THE DISTRIBUTION OF THE CORIOLIS PARAMETER IS ///) 0077
WRITE(6,170) F 0078
170 FORMAT(12F8.4) 0079
CALL GRAOUT(LV,LO,NOUT,F,NM,IA,JA,M2,N,BLANK) 0080
C C THE PROGRAM NOW READS IN TWO COMPONENTS OF THE NORMALIZED WIND 0081
STRESS. G1 GIVES ITS COMPONENT IN THE POSITIVE AXIS DIRECTION 0082
AND G2 GIVES ITS MAGNITUDE IN A DIRECTION 90 DEGREES TO THE LEFT 0083
THE STRESS HAS BEEN NONDIMENSIONALIZED USING \[ T = (\rho_f * F * S_1 / L) \]. 0084
C C TO SAVE SPACE ON THE INPUT CARDS THE EXPONENT PART OF THE STRESS 0085
IS READ IN FIRST 0086
DO 21 I = 1,NM
   T(I) = 0.0
   READ(5,112) I
112 FORMAT(13)
   FAC = 10.0**I
   READ(5,105) G1
   READ(5,105) G2
   DO 3 I = 1,NM
      G1(I) = FAC*G1(I)/H(I)
      G2(I) = FAC*G2(I)/H(I)
   3 HERE THE CURL OF T/H IS CALCULATED

   AREA = 2.598376
   FAC1 = 1.0/SQRT(3.)
   DO 5 K = 1,M2
      I = IA(K)
      J = JA(K)
   4 DO 4 L = I,J
      L1 = I+1
      L2 = L-N+1
      L3 = L-N
      L4 = L-1
      L5 = L+1
      L6 = L+N
   5 T(L) = 0.5*(G2(L1)-G2(L4)-(G1(L2)-G1(L6)+G1(L3)-G1(L5))*FAC1)

   WHILE IN THE LOOP RES(NM),HC1(NM) AND HC2(NM) ARE ALL CALCULATED
   FOR USE LATER IN THE RELAXATION OF THE STREAM FUNCTION AND THE
   BOTTOM STRESS TERM

   FAC = H(L)
   RES(L) = 1.0/(FAC+H(L1))+1.0/(FAC+H(L2))+1.0/(FAC+H(L3))
   +1.0/(FAC+H(L4))+1.0/(FAC+H(L5))+1.0/(FAC+H(L6))
   HC1(L) = 0.25*(1./H(L1) - 1./H(L4) + 0.5*(1./H(L2) - 1./H(L5)
      - 1./H(L3) + 1./H(L6))
   HC2(L) = (1./H(L2) - 1./H(L5) + 1./H(L3) - 1./H(L6)) * (3./8.)

   4 CONTINUE
   CONTINUE
5 CONTINUE
   WRITE(6,180)
   180 FORMAT(13)
   WRITE(6,173) T
   WRITE(6,118)
   CALL GRADOUT(LV,LO,NOUT,T,NM,IA,JA,M2,N,BLANK)

   THE ARRAY THAT CONTAINS THE INFORMATION NEEDED TO CALCULATE
   THE BOUNDARY VALUES FOR THE VORTICITY IS NOW READ IN.
C
READ(5,113) IBC
113 FORMAT(6(611,13))
C
RUN PARAMETERS ARE READ IN AT THIS POINT. C1 IS THE
NON-LINEAR COEFFICIENT S1/FDL**2; C2 IS THE FRICTIONAL
COEFFICIENT 2K/3FL**2; C3 IS THE BOTTOM STRESS
COEFFICIENT R/DF; DT IS THE NON-DIMENSIONAL TIME
STEP, TOUT IS THE INTERVAL AT WHICH DEPENDENT VARIABLES ARE
PRINTED OUT, TMAX IS THE MAXIMUM MODEL TIME, R IS THE
RELAXATION PARAMETER, CON IS THE RELATIVE CONVERGENCE FACTOR
FOR THE RELAXATION OF THE STREAM FUNCTION AND IPUNCH
IS A FLAG USED IF THE FINAL VALUE OF THE DEPENDENT
VARIABLES IS PUNCHED OUT AS A DATA DECK.
C
READ(5,114) C1,C2,C3,DT,TOUT,TMAX,R,CON,IPUNCH
114 FORMAT(6E10.3,F4.2,E10.3,I2)
WRITE(6,171)
171 FORMAT(1H1, 'THE RUN PARAMETERS C1,C2,C3,DT,TOUT,TMAX,R,CON AND
1 IPUNCH ARE' ///)
WRITE(6,172) C1,C2,C3,DT,TOUT,TMAX,R,CON,IPUNCH
172 FORMAT(8E14.3,16)
C
THE INITIAL VALUES FOR THE VORTICITY, STREAM FUNCTION AND
THE LOCAL CHANGE IN THE VORTICITY ONE TIME STEP BACK
ARE READ INTO THE PROGRAM
C
READ(5,115) S
READ(5,115) V
READ(5,115) G1
115 FORMAT(6E12.5)
C
THE TIME CONTROL IS STARTED NOW AND THE ACTUAL
CALCULATION OF THE PROGRAM BEGINS
C
ITER = 0
TIME = 0.0
6 CONTINUE
FAC = TIME/TOUT+0.01*DT
I = IFIX(FAC)
FAC = FLOAT(I)
FAC = TOUT*FAC
FAC = TIME-FAC
FAC = ABS(FAC)
DTERR = 0.1*DT
IF (FAC.GT.DTERR) GO TO 7
C
IN THIS SECTION THE DEPENDENT VARIABLES ARE PRINTED OUT
C
C
WRITE(6,116) TIME
WRITE(6,160)
160 FORMAT(24H THE VORTICITY FIELD IS ///)
WRITE(6,173) V
173 FORMAT(6E14.5)
WRITE(6,118)
CALL GRAGOUT(LV,LO,NOTU, V, NM, IA, JA, M2, N, BLANK)
WRITE(6,116) TIME
116 FORMAT(1H1, 43H THE NONDIMENSIONAL TIME INTO THE MODEL IS , E12.3 ///)
WRITE(6,117) ITER
117 FORMAT(39H THE VALUE OF THE STREAM FUNCTION AFTER , I8,
E 11H ITERATIONS ///)
WRITE(6,173) S
WRITE(6,118)
118 FORMAT(1H1)
CALL GRAGOUT(LV,LO,NOTU, S, NM, IA, JA, M2, N, BLANK)
7 CONTINUE
C
A MEASURE OF THE TOTAL KINETIC ENERGY OF THE FLOW
IS MONITORED AS A STABILITY CHECK.
C
FAC = C*0
DO 152 K = 1, M2
I = IA(K)
J = JA(K)
DO 151 L = I, J
A1 = S(L)
FAC = FAC + (S(L+1)-A1)**2 + (S(L-N+1)-A1)**2 + (S(L-N)-A1)**2
E + (S(L-1)-A1)**2 + (S(L+N-1)-A1)**2 + (S(L+N)-A1)**2
151 CONTINUE
152 CONTINUE
WRITE(6,153) TIME, FAC
153 FORMAT(14H THE TIME IS , F10.4, 22H THE KINETIC ENERGY IS, E14.3)
WRITE(6,379) VORT
379 FORMAT(25H THE TOTAL VORTICITY IS , E14.3)
TIME = TIME + DT
IF(TIME.GT.TMAX) GO TO 16
C
THE LOCAL TIME CHANGE IN THE VORTICITY AT THE PRESENT
TIME STEP IS NOW CALCULATED FOR ALL INTERNAL POINTS
C
DO 10 K = 1, M2
I = IA(K)
J = JA(K)
DO 9 L = I, J
L1 = L+1
10 CONTINUE
9 CONTINUE
L2 = L-N+1
L3 = L-N
L4 = L-1
L5 = L+N-1
L6 = L+N

HERE THE CONTRIBUTION FROM THE ADVECTION OF POTENTIAL VORTICITY IS CALCULATED

A1 = (C1*V(L1)+F(L1))/H(L1)
A2 = (C1*V(L2)+F(L2))/H(L2)
A3 = (C1*V(L3)+F(L3))/H(L3)
A4 = (C1*V(L4)+F(L4))/H(L4)
A5 = (C1*V(L5)+F(L5))/H(L5)
A6 = (C1*V(L6)+F(L6))/H(L6)

FAC = ((S(L2)-S(L1))*(A2+A1)+(S(L3)-S(L2))*(A3+A2)
E + (S(L4)-S(L3))*(A4+A3)+(S(L5)-S(L4))*(A5+A4)+(S(L6)-S(L5))
E *(A6+A5)+(S(L1)-S(L6))*(A1+A6))/(2.0*AREA)

THE CONTRIBUTION FROM THE DIFFUSION OF VORTICITY IS NOW CALCULATED AND ADDED ON

FAC = FAC+C2*(V(L1)+V(L2)+V(L3)+V(L4)+V(L5)+V(L6)-6.0*V(L))

THE CONTRIBUTION FROM THE CURL OF THE WIND IS ADDED ON

FAC = FAC + T(L)

THE CONTRIBUTION OF THE BOTTOM STRESS IS ADDED ON AND THE RESULT STORED IN G2(L)

G2(L) = FAC - (C3/H(L1)) * (V(L) + (HC1(L) * (S(L1) - S(L4) + 0.5
1 * (S(L2) - S(L5) - S(L3) + S(L6)))) + (HC2(L) * (S(L2) - S(L5)
2 + S(L3) - S(L6))))

CONTINUE

THE NEW BOUNDARY VALUES FOR THE VORTICITY ARE CALCULATED IN THIS SECTION

FIRST THE LOCAL CHANGE OF VORTICITY IS CALCULATED FOR ALL BOUNDARY POINTS EXCEPT WHERE INFLOW OCCURS, THOSE VALUES DO NOT CHANGE WITH TIME.

DO 8 I = 1,NB
L = IBC(7,I)
LA(1) = L+1
LA(2) = L+N+1
LA(3) = L-N

8 CONTINUE
LA(4) = L-1
LA(5) = L+N-1
LA(6) = L+N
FAC = 0.0
TRI = 1.0
DO 84 J = 1,6
K = J+1
IF(K.EQ.7) K = 1
M = J-1
IF(M.EQ.0) M = 6

C HERE THE CONTRIBUTION FROM ADVECTION IS CALCULATED IF THE TRIANGLE
C IS INTERIOR TO THE MODEL
C
IF(IBC(J,I).EQ.0 OR IBC(K,I).EQ.0) GO TO 81
L1 = LA(J)
L2 = LA(K)
FAC = FAC + (S(L2)-S(L1))*((C1*V(L2)+F(L2))/H(L2) +
E
(C1*V(L1)+F(L1))/H(L1))/(2.0*AREA)

C HERE THE CONTRIBUTION FROM DIFFUSION PARALLEL THE BOUNDARY IS
C CALCULATED
C
81 IF(IBC(J,I).NE.3 AND IBC(J,I).NE.4) GO TO 811
L1 = LA(J)
FAC = FAC + C2*(V(L1)-V(L))/2.0

C HERE THE CONTRIBUTION OF DIFFUSION FROM THE INTERIOR IS
C CALCULATED
C
811 IF(IBC(J,I).NE.1 AND IBC(J,I).NE.2) GO TO 812
L1 = LA(J)
FAC = FAC + C2*(V(L1)-V(L))

812 CONTINUE

C HERE THE DIFFUSION NORMAL TO THE BOUNDARY IS CALCULATED IF A NO
C SLIP CONDITION IS USED. NOTE LENGTH FACTORS CANCEL OUT, NO EXTRA
C COEFFICIENTS ARE NEEDED.
C
IF(IBC(J,I).NE.2) GO TO 82
IF(IBC(M,I).EQ.2 AND IBC(K,I).EQ.2) GO TO 82
IF(IBC(M,I).EQ.2) GO TO 813
L1 = LA(J)
L2 = LA(M)
FAC = FAC + C2*(V(L1)-0.5*(V(L2)+V(L)))

813 IF(IBC(K,I).EQ.2) GO TO 82
L1 = LA(J)
L2 = LA(K)
FAC = FAC + C2*(V(L1)-0.5*(V(L2)+V(L)))

NOW THE POSSIBILITY OF VORTICITY BEING ADVECTED OUT OF THE MODEL IS TAKEN CARE OF.

82 IF(IBC(J,I).NE.4) GO TO 83
L1 = LA(J)
FAC = FAC - ABS(S(L1)-S(L))*(C1*V(L1)+F(L1))/H(L1)+(C1*V(L) + F(L))/H(L))*0.5/AREA

HERE THE NUMBER OF OUTSIDE POINTS ARE COUNTED FOR THE AREA CORRECTION

83 IF(IBC(J,I).EQ.0) TRI = TRI + 1.0
84 CONTINUE

NOW THE LOCAL CHANGE IS CORRECTED FOR AREA

G2(L) = FAC*6.0/(6.0-TRI)
8 CONTINUE

THE NEW BOUNDARY VALUE FOR THE VORTICITY IS CALCULATED USING THE ADAMS-BASHFORTH SCHEME AND G2 IS TRANSFERED TO G1

DO 80 I = 1,NB
L = IBC(7,I)
V(L) = V(L) + DT*(1.5*G2(L)-0.5*G1(L))
TRI = 1.0
DO 888 J = 1,6
888 IF(IBC(J,I).EQ.0) TRI = TRI + 1.0
CONTINUE
VORT = VORT + V(L)*(6.0-TRI)/H(L)

G1(L) = G2(L)

THE NEW VALUE OF THE VORTICITY IS CALCULATED AT ALL INTERIOR POINTS USING THE ADAMS-BASHFORTH METHOD AND THE PRESENT VALUE OF THE LOCAL CHANGE IS TRANSFERED FROM G2 TO G1 TO BE USED IN THE NEXT TIME STEP

DO 12 K = 1,M2
I = IA(K)
J = JA(K)
DO 11 L = I,J
V(L) = V(L)+DT*(1.5*G2(L)-0.5*G1(L))
VORT = VORT + V(L)*6.0/H(L)

11 G1(L) = G2(L)
12 CONTINUE
THE NEW VALUE OF THE STREAM FUNCTION IS OBTAINED
VIA THE RELAXATION OF THE NEW VORTICITY FIELD. A
SWEEP PROCEDURE IS STARTED AND 100 SWEEPS IS SET AS
THE UPPER LIMIT ON PASSES THROUGH THE GRID.

ITER = 100
DO 15 II = 1, 100
ERR = 0.0
DO 14 K = 1, M2
   I = IA(K)
   J = JA(K)
   DO 13 L = I, J
   L1 = L+1
   L2 = L-N+1
   L3 = L-N
   L4 = L-1
   L5 = L+N-1
   L6 = L+N
   HO = H(L)
   FAC = 1.33333*(S(L1)/(H(L1)+HO)+S(L2)/(H(L2)+HO))
   +S(L3)/(H(L3)+HO)+S(L4)/(H(L4)+HO)+S(L5)/(H(L5)+HO)
   +S(L6)/(H(L6)+HO)-S(L)*RES(L))-V(L)
   FAC = FAC/(1.33333*RES(L))
   S(L) = S(L)+R*FAC
   FAC = ABS(FAC)
   IF(FAC.GT.ERR) ERR = FAC
13 CONTINUE
14 CONTINUE
15 CONTINUE
16 CONTINUE
THE CONVERGENCE IS NOW CHECKED AND IF THE MAX
RESIDUE IS LESS THEN CON THE ITERATION LOOP IS EXITED

IF(1.0(CON) GO TO 15
ITER = II
GO TO 155
155 CONTINUE
GO TO 6
16 CONTINUE
IF(IPUNCH.EQ.0) GO TO 17
WRITE(7,115) S
WRITE(7,115) V
WRITE(7,115) G1
17 CONTINUE
STOP
END
SUBROUTINE GRAOUT(LV,LO,NOUT,A,NM,IA,JA,M2,N,BLANK)
DIMENSION LV(20),LO(NOUT),A(NM),IA(M2),JA(M2)

THIS SUBROUTINE TAKES THE INTERIOR VALUES IN ARRAY A SCALES
THEM, TRANSFORMS THEM TO AN ALPHA CODE AND PRINTS
THEM OUT IN A CARTESIAN GRID

M = NM/N
DO 1 K = 1,NOUT
   1 LO(K) = BLANK
   K = IA(1)
   AMAX = A(K)
   AMIN = AMAX
   DO 3 K = 1,M2
      I = IA(K)
      J = JA(K)
      DO 2 L = I,J
      2 IF(A(L).GT.AMAX) AMAX = A(L)
      IF(A(L).LT.AMIN) AMIN = A(L)
      CONTINUE
      ADIF = AMAX-AMIN
      IF(ADIF.EQ.0.0) GO TO 8
      DO 5 K = 1,M2
         I = IA(K)
         J = JA(K)
         DO 4 L = I,J
            LL = 2*L-1+(L-1)/N*(M-1)
            NN = TRUNC(19.999*(AMAX-A(L))/ADIF)
            4 LO(LL) = LV(NN+1)
         CONTINUE
      WRITE(6,6) LO
   CONTINUE
   WRITE(6,6) LO
   DO 8 K = 1,NOUT
      WRITE(6,9) FORMAT(1H0,19A2)
      RETURN
     8 FORMAT(6,9)
      WRITE(6,9) FORMAT(47H THE ARRAY HAD A CONSTANT VALUE NO GRAPH GIVEN )
      RETURN
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6
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A numerical ocean model driven by surface stress and a source-sink distribution is developed for a homogeneous ocean. Non-linearities, lateral friction and bottom friction are included. The basin shape can be varied to accommodate a large variety of configurations. Variable bathymetry and sources/sinks around the perimeter are included. The numerical scheme is conditionally stable and has second order accuracy in space and time.

A number of test cases are run to explore the dynamic significances of the various processes represented. The possible influence of these processes on the circulation of the Arctic ocean are discussed.
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